A Logic with a Conditional Probability Operator

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Overview

Outline

History overview

- Probabilistic logics allow for strict reasoning *about* probabilities using well-defined syntax and semantics
- The formulas in these logics remain either true or false
- Keisler: Probabilistic quantors (mid 70's)
- Nilsson, N.: Probabilistic logic. Articial intelligence 28, 7187 (1986)
- Serbia: Miodrag Rašković

Probabilistic logics

Logic with a probabilistic operator - Probabilistic logic

- $P_{\geq s}$, where $s \in S \subset [0, 1]$
- $(P_{\geq s}\alpha \wedge P_{\geq t}(\alpha \to \beta)) \to P_{\geq r}\beta$
- Kripke models with probability measures defined over worlds

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Logic with a conditional probability operator

- New operator $CP(\alpha, \beta)$
- Kolmogorov-style definition for conditional probabilities: $P(\alpha|\beta) = \frac{P(\alpha \wedge \beta)}{P(\beta)}, P(\beta) > 0$

Motivation

R. Fagin, J. Halpern, N. Megiddo. Logic for reasoning about probabilities.

- Logic for linear weight formulas (LWF)

$$w(\alpha) + 3w(\beta) - 5w(\gamma) \geqslant 0.2$$

weak completeness + decidability (NP)

- Logic for polynomial weight formulas (PWF)

$$w(\alpha)^2 w(\beta) - 5w(\gamma) \geqslant w(\alpha)w(\gamma)$$

decidability (PSPACE)

- Interpretation of polynomial weight formulas in first order logic

$$\forall x(xw(\alpha)^2 + w(\beta) = 0.7)$$

weak completeness + decidability (EXPSPACE)

Goals for LPCP

- Intermediate logic (with respect to LWF and PWF)
- Strong completness (every consistent set of formulas is satisfiable)
- Decidablity (PSPACE)

Outline

- $Var = \{p_n \mid n < \omega\}$ is the set of propositional variables.

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Definition

The set *Term* of all probabilistic terms is recursively defined as follows:

- $Term(0) = \{\underline{s} \mid s \in \mathbb{Q}\} \cup \{CP(\alpha, \beta) \mid \alpha, \beta \in For_C\}$.
- $Term(n+1) = Term(n) \cup \{(\mathtt{f} + \mathtt{g}), (\underline{s} \cdot \mathtt{g}), (-\mathtt{f}) \mid \mathtt{f}, \mathtt{g} \in Term(n),\ s \in \mathbb{Q}\}$
- $Term = \bigcup_{n=0}^{\infty} Term(n)$. □

- $\mathtt{f}, \mathtt{g}$ and $\mathtt{h}$: variables used for denoting terms
- $\mathtt{f} + \mathtt{g} = (\mathtt{f} + \mathtt{g})$
- $\mathtt{f} + \mathtt{g} + \mathtt{h} = ((\mathtt{f} + \mathtt{g}) + \mathtt{h})$
- $-\mathtt{f} = (-\mathtt{f})$
- $\mathtt{f} - \mathtt{g} = (\mathtt{f} + (-\mathtt{g}))$

To simplify notation, we will write $P(\alpha)$ instead of $CP(\alpha, \top)$, where $\top$ is an arbitrary tautology instance.

Definition

A basic probabilistic formula is any formula of the form $\mathtt{f} \geqslant \underline{0}$. Furthermore, we define the following abbreviations:

- $\mathtt{f} \leqslant \underline{0}$ is $-\mathtt{f} \geqslant \underline{0}$;
- $\mathtt{f} = \underline{0}$ is $\mathtt{f} \leqslant \underline{0} \ \wedge \ \mathtt{f} \geqslant \underline{0}$;
- $\mathtt{f} > \underline{0}$ is $\neg(\mathtt{f} \leqslant \underline{0})$;
- $\mathtt{f} \neq \underline{0}$ is $\neg(\mathtt{f} = \underline{0})$;
- $\mathtt{f} < \underline{0}$ is $\neg(\mathtt{f} \geqslant \underline{0})$;
- $\mathtt{f} \geqslant \mathtt{g}$ is $\mathtt{f} - \mathtt{g} \geqslant \underline{0}$.

We define $\mathtt{f} \leqslant \mathtt{g}$, $\mathtt{f} > \mathtt{g}$, $\mathtt{f} < \mathtt{g}$, $\mathtt{f} = \mathtt{g}$ and $\mathtt{f} \neq \mathtt{g}$ in a similar way. □

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- For_P denotes the set of all probabilistic formulas.
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- $For = For_C \cup For_P$
- Φ, Ψ and Θ: variables used for denoting elements of For

Outline

We define the notion of a model as a special kind of Kripke model. Namely, *a model M* is any tuple $\langle W, H, \mu, v \rangle$ such that:

- W is a nonempty set. As usual, its elements will be called worlds.
- H is an algebra of sets over W.
- $\mu : H \longrightarrow [0,1]$ is a finitely additive probability measure.
- $v : For_C \times W \longrightarrow \{0,1\}$ is a truth assignment.
- $[\alpha]_M = \{w \in W \mid v(\alpha, w) = 1\}$

We say that M is *measurable* if $[\alpha]_M \in H$ for all $\alpha \in For_C$.

Definition

Let $M = \langle W, H, \mu, v\rangle$ be any measurable model. We define the satisfiability relation $\models$ recursively as follows:

- $M \models \alpha$ if $v(\alpha, w) = 1$ for all $w \in W$.
- $M \models \mathtt{f} \geqslant \underline{0}$ if $\mathtt{f}^M \geqslant 0$, where $\mathtt{f}^M$ is recursively defined in the following way:
  - $\underline{s}^M = s$.
  - $CP(\alpha, \beta)^M = \mu([\alpha \wedge \beta]) \cdot \mu([\beta])^{-1}$.
  - $(\mathtt{f} + \mathtt{g})^M = \mathtt{f}^M + \mathtt{g}^M$.
  - $(\underline{s} \cdot \mathtt{g})^M = s \cdot \mathtt{g}^M$.
  - $(-\mathtt{f})^M = -(\mathtt{f}^M)$.
- $M \models \neg\phi$ if $M \not\models \phi$.
- $M \models \phi \wedge \psi$ if $M \models \phi$ and $M \models \psi$. □

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- Φ is *valid* if it is satisfied in every measurable model.
- The set T of formulas is *satisfiable* if there is a measurable model M such that $M \models \Phi$ for all $\Phi \in T$.

Outline

AX_{LPCP}

- Axioms for propositional reasoning
- Axioms for probabilistic reasoning
- Arithmetical axioms
- Inference rules

Axioms for propositional reasoning

A1. $\tau(\Phi_1, \ldots, \Phi_n)$, where $\tau(p_1, \ldots, p_n) \in For_C$ is any tautology and Φ_i are either all propositional or all probabilistic.

Axioms for probabilistic reasoning

A2. $P(\alpha) \geqslant \underline{0}$;
A3. $P(\top) = \underline{1}$;
A4. $P(\bot) = \underline{0}$;
A5. $P(\alpha \leftrightarrow \beta) = \underline{1} \ \rightarrow \ P(\alpha) = P(\beta)$;
A6. $P(\alpha \vee \beta) = P(\alpha) + P(\beta) - P(\alpha \wedge \beta)$;
A7. $(P(\alpha \wedge \beta) = \underline{r} \wedge P(\beta) = \underline{s}) \rightarrow CP(\alpha, \beta) = \underline{r \cdot s^{-1}}, \underline{s} > 0$.

Arithmetical axioms

A8. $\underline{r} \geqslant \underline{s}$, whenever $r \geqslant s$;

A9. $\underline{s} \cdot \underline{r} = \underline{sr}$;

A10. $\underline{s} + \underline{r} = \underline{s + r}$;

A11. $\mathtt{f} + \mathtt{g} = \mathtt{g} + \mathtt{f}$;

A12. $(\mathtt{f} + \mathtt{g}) + \mathtt{h} = \mathtt{f} + (\mathtt{g} + \mathtt{h})$;

A13. $\mathtt{f} + \underline{0} = \mathtt{f}$;

A14. $\mathtt{f} - \mathtt{f} = \underline{0}$;

A15. $(\underline{r} \cdot \mathtt{f}) + (\underline{s} \cdot f) = \underline{r + s} \cdot \mathtt{f}$;

A16. $\underline{s} \cdot (\mathtt{f} + \mathtt{g}) = (\underline{s} \cdot \mathtt{f}) + (\underline{s} \cdot \mathtt{g})$

A17. $\underline{r} \cdot (\underline{s} \cdot \mathtt{f}) = \underline{r \cdot s} \cdot \mathtt{f}$

A18. $\underline{1} \cdot \mathtt{f} = \mathtt{f}$

A19. $\mathtt{f} \geqslant \mathtt{g} \ \vee \ \mathtt{g} \geqslant \mathtt{f}$

A20. $(\mathtt{f} \geqslant \mathtt{g} \ \wedge \ \mathtt{g} \geqslant \mathtt{h}) \rightarrow \mathtt{f} \geqslant \mathtt{h}$

A21. $\mathtt{f} \geqslant \mathtt{g} \ \rightarrow \ \mathtt{f} + \mathtt{h} \geqslant \mathtt{g} + \mathtt{h}$

A22. $(\mathtt{f} \geqslant \mathtt{g} \ \wedge \ \underline{s} > \underline{0}) \ \rightarrow \underline{s} \cdot \mathtt{f} \geqslant \underline{s} \cdot \mathtt{g}$

Inference rules

R1. From Φ and $\Phi \rightarrow \Psi$ infer Ψ.

R2. From α infer $P(\alpha) = \underline{1}$.

R3. From the set of premises $\{\phi \rightarrow \mathtt{f} \geqslant \underline{-n^{-1}} \mid n = 1, 2, 3, \ldots\}$ infer $\phi \rightarrow \mathtt{f} \geqslant \underline{0}$.

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Notions of theorem and consistency are defined as usual. The only difference is in the fact that the length of a proof may be countable.

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- We will begin with Deduction theorem and some auxiliary statements
- Then, we will describe how a consistent set T of sentences can be extended to a suitable maximal consistent set
- After that, we will show how a canonical model can be constructed out of such maximal consistent sets
- Finally, we prove that for every world w from the canonical model, a sentence A is satisfied in w if and only if $A \in w$, and as a consequence we obtain that the set T is satisfiable

Theorem (Deduction theorem)

Suppose that T is an arbitrary set of formulas and that $\Phi, \Psi \in For$. Then, $T \vdash \Phi \rightarrow \Psi$ iff $T \cup \{\Phi\} \vdash \Psi$.

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Lemma

Suppose that T is a consistent set of formulas. If $T \cup \{\phi \to \mathtt{f} \geqslant \underline{0}\}$ is inconsistent, then there is a positive integer n such that $T \cup \{\phi \to \mathtt{f} < \underline{-n^{-1}}\}$ is consistent.

Definition

Suppose that T is a consistent set of formulas and that $For_P = \{\phi_i \mid i = 0, 1, 2, 3, \dots\}$. We define a completion T^* of T recursively as follows:

1. $T_0 = T \cup \{\alpha \in For_C \mid T \vdash \alpha\} \cup \{P(\alpha) = \underline{1} \mid T \vdash \alpha\}$.
2. If $T_i \cup \{\phi_i\}$ is consistent, then $T_{i+1} = T_i \cup \{\phi_i\}$.
3. If $T_i \cup \{\phi_i\}$ is inconsistent, then:
   1. If ϕ_i has the form $\psi \to \mathtt{f} \geqslant \underline{0}$, then $T_{i+1} = T_i \cup \{\psi \to \mathtt{f} < \underline{-n^{-1}}\}$, where n is a positive integer such that T_{i+1} is consistent.
   2. Otherwise, $T_{i+1} = T_i$.

□

Theorem

Suppose that T is a consistent set of formulas and that T^* is constructed as above. Then:

1. *T^* is deductively closed, id est, $T^* \vdash \Phi$ implies $\Phi \in T^*$.*
2. *There is $\phi \in For_P$ such that $\phi \notin T^*$.*
3. *For each $\phi \in For_P$, either $\phi \in T^*$, or $\neg\phi \in T^*$.*

Canonical Model

For the given completion T^*, we define a *canonical model* M^* as follows:

- W is the set of all functions $w : For_C \longrightarrow \{0, 1\}$ with the following properties:
  - w is compatible with $\neg$ and $\wedge$.
  - $w(\alpha) = 1$ for each $\alpha \in T^*$.
- $v : For_C \times W \longrightarrow \{0, 1\}$ is defined by $v(\alpha, w) = 1$ iff $w(\alpha) = 1$.
- $H = \{[\alpha] \mid \alpha \in For_C\}$.
- $\mu : H \longrightarrow [0, 1]$ is defined by
  $\mu([\alpha]) = \sup\{s \in [0, 1] \cap \mathbb{Q} \mid T^* \vdash P(\alpha) \geqslant \underline{s}\}$.

Lemma

M^* is a measurable model.

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Theorem (Strong completeness theorem)

Every consistent set of formulas has a measurable model.

Outline

Theorem

Satisfiability of probabilistic formulas is decidable and it is decidable in PSPACE.

Rewriting to PWF in linear time:

$$CP(\alpha, \beta) \equiv \frac{w(\alpha \wedge \beta)}{w(\beta)}$$

Example ($CP(\alpha, \beta) + CP(\alpha, \gamma) \geqslant \frac{1}{2}$)

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Example $(CP(\alpha,\beta) + CP(\alpha,\gamma) \geqslant \frac{1}{2})$

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$\alpha \wedge \beta \Leftrightarrow (\alpha \wedge \beta \wedge \gamma) \vee (\alpha \wedge \beta \wedge \neg\gamma)$
$\alpha \wedge \gamma \Leftrightarrow (\alpha \wedge \beta \wedge \gamma) \vee (\alpha \wedge \neg\beta \wedge \gamma)$
$\beta \Leftrightarrow$
$(\alpha\wedge\beta\wedge\gamma)\vee(\alpha\wedge\neg\beta\wedge\neg\gamma)\vee(\neg\alpha\wedge\beta\wedge\gamma)\vee(\neg\alpha\wedge\beta\wedge\neg\gamma)$
$\gamma \Leftrightarrow$
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$x_1 = P(\alpha \wedge \beta \wedge \gamma)$
$x_2 = P(\alpha \wedge \beta \wedge \neg\gamma)$
$x_3 = P(\alpha \wedge \neg\beta \wedge \gamma)$
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$x_5 = P(\neg\alpha \wedge \beta \wedge \gamma)$
$x_6 = P(\neg\alpha \wedge \beta \wedge \neg\gamma)$
$x_7 = P(\neg\alpha \wedge \neg\beta \wedge \gamma)$
$x_8 = P(\neg\alpha \wedge \neg\beta \wedge \neg\gamma)$

Example $(CP(\alpha,\beta) + CP(\alpha,\gamma) \geqslant \frac{1}{2})$

$$\frac{P(\alpha \wedge \beta)}{P(\beta)} + \frac{P(\alpha \wedge \gamma)}{P(\gamma)} \geqslant \frac{1}{2}$$

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$$
\begin{aligned}
x_1 &= P(\alpha \wedge \beta \wedge \gamma) \\
x_2 &= P(\alpha \wedge \beta \wedge \neg\gamma) \\
x_3 &= P(\alpha \wedge \neg\beta \wedge \gamma) \\
x_4 &= P(\alpha \wedge \neg\beta \wedge \neg\gamma) \\
x_5 &= P(\neg\alpha \wedge \beta \wedge \gamma) \\
x_6 &= P(\neg\alpha \wedge \beta \wedge \neg\gamma) \\
x_7 &= P(\neg\alpha \wedge \neg\beta \wedge \gamma) \\
x_8 &= P(\neg\alpha \wedge \neg\beta \wedge \neg\gamma)
\end{aligned}
$$

$$\exists x_1 \dots x_8 \left(\bigwedge_{i=1}^{8} x_i \geqslant 0 \wedge \sum_{i=1}^{8} x_i = 1 \wedge \frac{x_1 + x_2}{x_1 + x_2 + x_5 + x_7} + \frac{x_1 + x_3}{x_1 + x_3 + x_5 + x_7} \geqslant \frac{1}{2} \right)$$

Outline

- Zoran Ognjanović, Jozef Kratica, Miloš Milovanović, A genetic algorithm for satisfiability problem in a probabilistic logic: A first report, 2001.
- Zoran Ognjanović, Uroš Midić, Jozef Kratica, A genetic algorithm for probabilistic SAT problem, 2004.
- Zoran Ognjanović, Uroš Midić, Nenad Mladenović, A Hybrid Genetic and Variable Neighborhood Descent for Probabilistic SAT Problem, 2005.
- Dejan Jovanović, Nenad Mladenović, Zoran Ognjanović, Variable Neighborhood Search for the Probabilistic Satisfiability Problem, 2007

- To be done: Checker for LPCP (VNS and parallelization)

Contact information

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