# Functional Programming with Isabelle/HOL

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## Overview

Viewing *Isabelle/HOL* as a functional programming language:

1. *Isabelle/HOL* Specification Tools.
2. Code Generation from *Isabelle/HOL*-Theories.
3. Behind the Scene.

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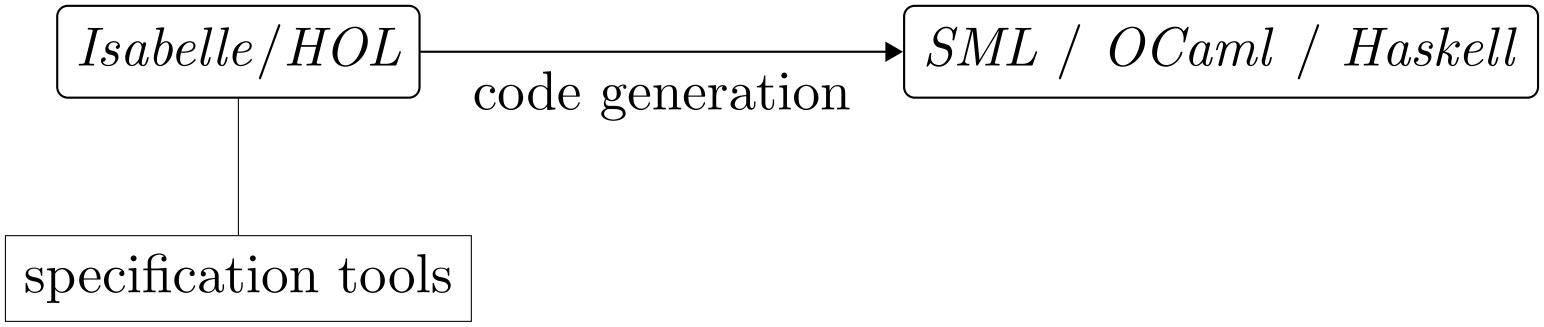

## $Isabelle/HOL$ **specification tools**

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  $\mathit{nonsense\text{-}def}$: $\mathit{nonsense}\ n = \mathit{Suc}\ (\mathit{nonsense}\ n)$

**lemma** $0 = \mathit{Suc}\ 0$
**proof** $-$
  **from** $\mathit{nonsense\text{-}def}$
    **have** $\mathit{nonsense}\ 0 - \mathit{nonsense}\ 0 = \mathit{Suc}\ (\mathit{nonsense}\ 0) - \mathit{nonsense}\ 0$ **by** $\mathit{simp}$
  **then show** $0 = \mathit{Suc}\ 0$ **by** $\mathit{simp}$
**qed**

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axiomatization nonsense :: nat ⇒ nat where
  nonsense-def: nonsense n = Suc (nonsense n)

lemma 0 = Suc 0
proof −
  from nonsense-def
    have nonsense 0 − nonsense 0 = Suc (nonsense 0) − nonsense 0 by simp
  then show 0 = Suc 0 by simp
qed
```

Things have to be properly *constructed*, that is:

- Find an appropriate *primitive definition*.
- *Derive* desired specification (*honest toil*).

*Specification tools* automate this.

## The *Isabelle/HOL* toolbox

$$\begin{tikzcd}
\boxed{\textit{Isabelle/HOL}} \arrow[r, "\text{code generation}"'] \arrow[d, dash] & \boxed{\textit{SML / OCaml / Haskell}} \\
\boxed{\text{specification tools}} &
\end{tikzcd}$$

***inductive predicates*** Knaster-Tarski fixed point theorem

***inductive datatypes*** inductive predicate *plus* *typedef*

***primitive recursion*** primitive recursion combinator

***terminating functions*** explicit function graph *plus* definite choice

## Type classes

Leightweight mechanism for *overloading* plus *abstract specification*.

Example: *algebra*

## Code generator basics

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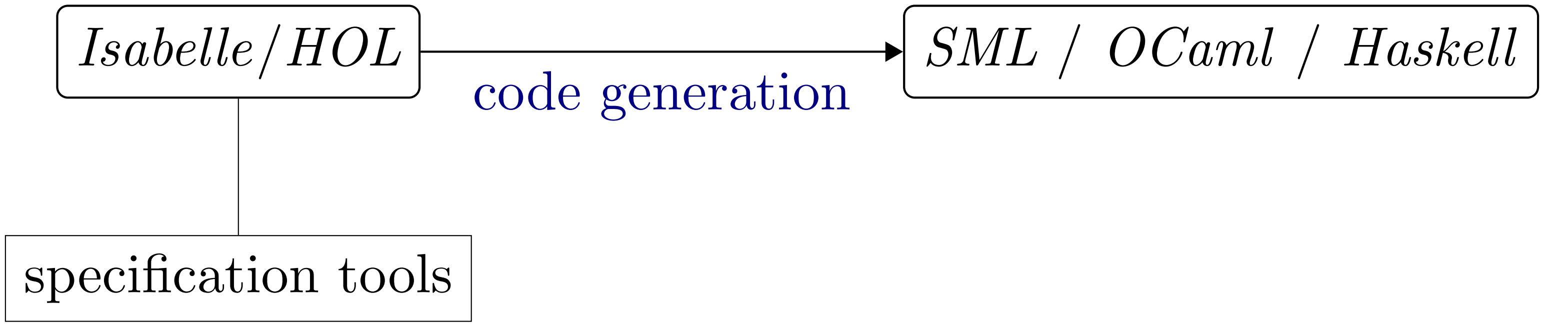

## Code generation using shallow embedding

Correctness criterion: semantics of generated target language program $P$ describes a *term rewrite system* where each derivation can be *simulated* in the theory $\Theta$ of the logic:

```
sum [Suc Zero_nat, Suc Zero_nat]
```

```
datatype nat = Suc of nat | Zero_nat;

fun plus_nat (Suc m) n = plus_nat m (Suc n)
  | plus_nat Zero_nat n = n;

fun sum [] = Zero_nat
  | sum (m :: ms) = plus_nat m (sum ms);
```

```
Suc (Suc Zero_nat)
```

$$\begin{tikzcd}
\mathrm{t} \arrow[rr, Leftrightarrow, "\text{identification}"] \arrow[d, dash] & & \mathrm{t} \arrow[d, dash] \\
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(partial correctness)

## Examples

- *amortised queues*
- *amortised queues* with poor man's datatype abstraction
- *algebra* with type classes

## A closer look at code generation

## How does a code generator look like?

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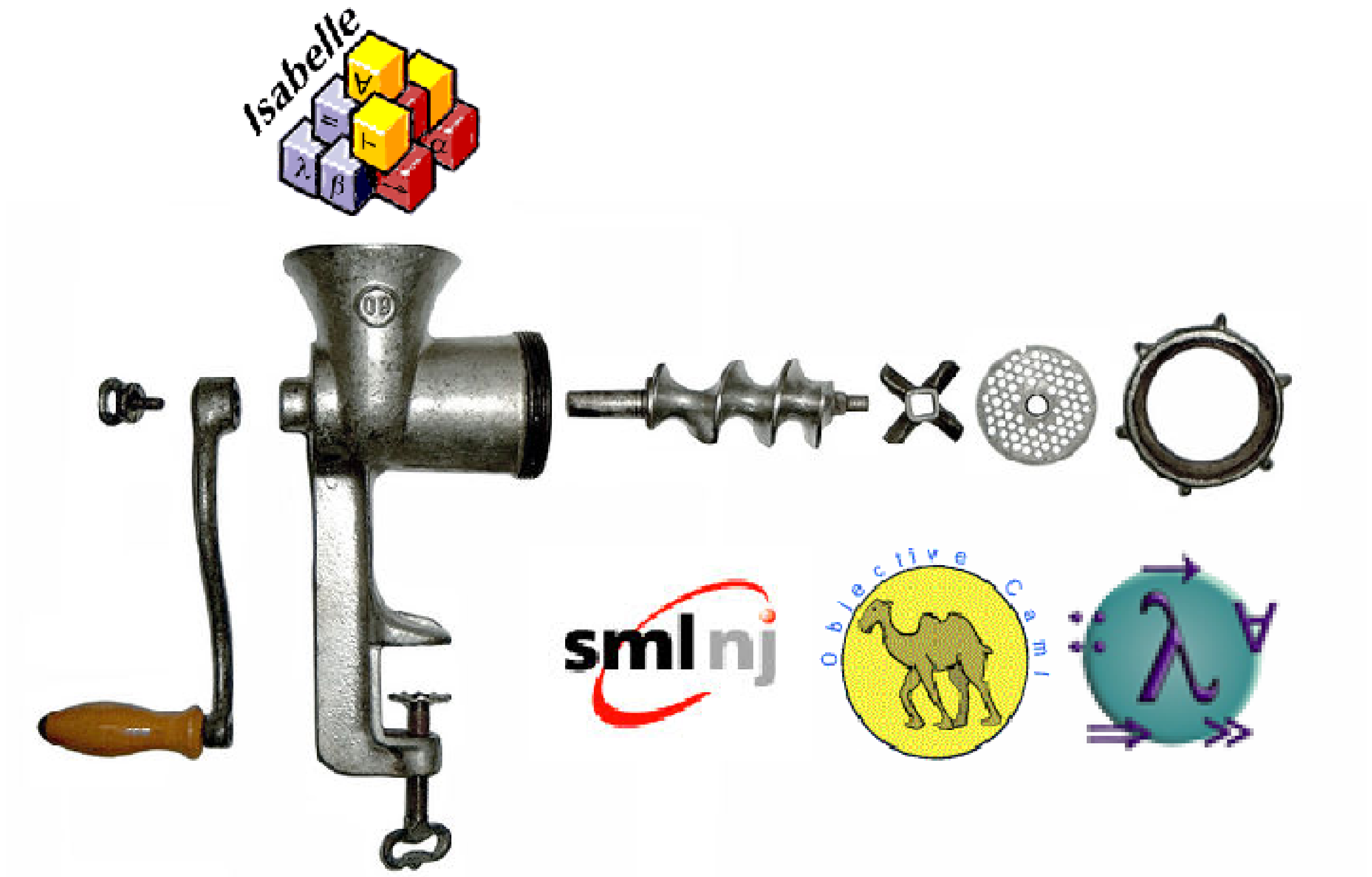
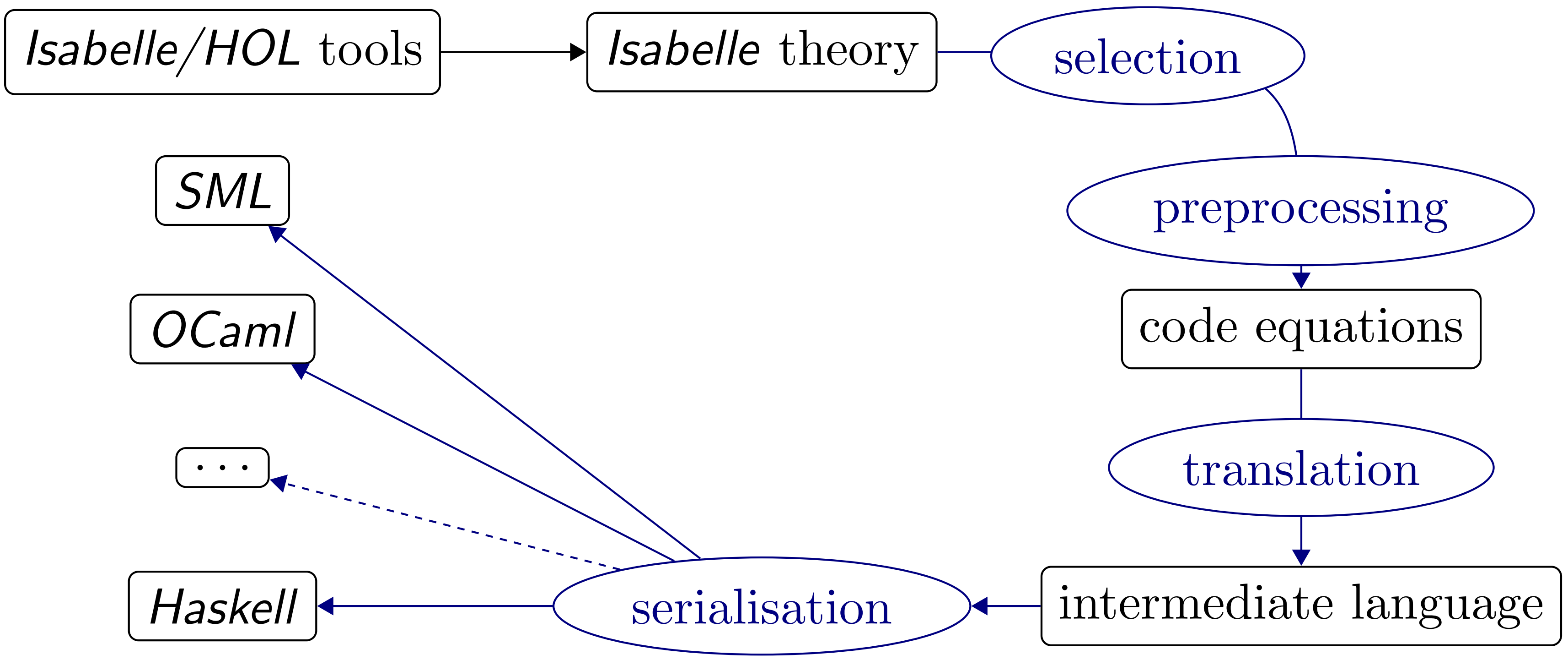


## Architecture



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$$\mathit{data}\ \kappa\ \overline{\alpha}_k = f_1\ \mathit{of}\ \overline{\tau_1} \mid \ldots \mid f_n\ \mathit{of}\ \overline{\tau_n}$$

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$$\begin{array}{l} \mathit{class}\ c \subseteq c_1 \cap \ldots \cap c_m\ \mathit{where} \\ \quad f_1 :: \forall\,\alpha.\ \tau_1,\ \ldots,\ f_n :: \forall\,\alpha.\ \tau_n \end{array}$$

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…a kind of "Mini-Haskell"

…not "All-gol", but "Thin-gol"

## Selecting

Two degrees of freedom:

**code equations**
*by default:* **definition**, **primrec**, **fun**, **function**
*explicitly:* attribute [*code*]

**datatype constructors**
*by default:* **datatype**, **record**
*explicitly:* **code-datatype**

## Preprocessing

Interface to plugin arbitrary theorem transformations:

**rewrites**

```
simpset
```

**function transformators**

```
theory -> thm list -> thm list
```

## Serialising

Adaption to target-language specifics:

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$\cup$ {serialisation}

## What remains

**Not mentioned here**

- implementing equality
- code extraction from proofs

**Ongoing work and research**

- turning inductive predicates into equations
- *Haskabelle*: importing *Haskell* files
- Quickcheck
- concept for datatype abstraction

**Further reading**

- Tutorials in the *Isabelle* distribution for functions, code generation etc.
- PhD thesis on code generation (under heavy construction. . . )

. . .

**Happy proving, happy hacking**
**Thanks for your attention**