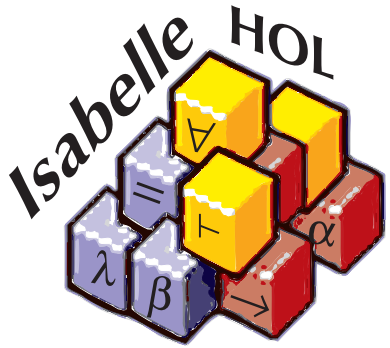
New trends in code generation: Haskabelle and the Predicate Compiler



Florian Haftmann
joint work with Stefan Berghofer and Lukas Bulwahn
Technische Universität München

Workshop in Belgrade – Jan. 2010

Part one: Haskabelle

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- a realistic example: finite maps
  - taken from http://hackage.haskell.org/cgi-bin/hackage-scr package/FiniteMap-0.1
    ▶ EXAMPLE

Adaptation

Excerpt from `default/adapt.txt` in the Haskabelle distribution:

```
classes
  "Prelude.Eq"          "Prelude.eq"
types
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Purpose:

- provide counterparts for fundamental Haskell definitions
- reuse existing Isabelle definitions (proofs!)

State of the art

What we have

- promising preliminary experiments with the seL4 operating system kernel http://ertos.nicta.com.au/research/l4.verified/
- used by secunet http://www.secunet.com/en/ "IT security beyond expectations"

What we encourage

- give it a try
- extend Haskabelle to fit your needs

Part two: Predicate Compiler

Executing Inductive Predicates in a functional language

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- Type systems
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Important for Prototypes and Counterexample generation

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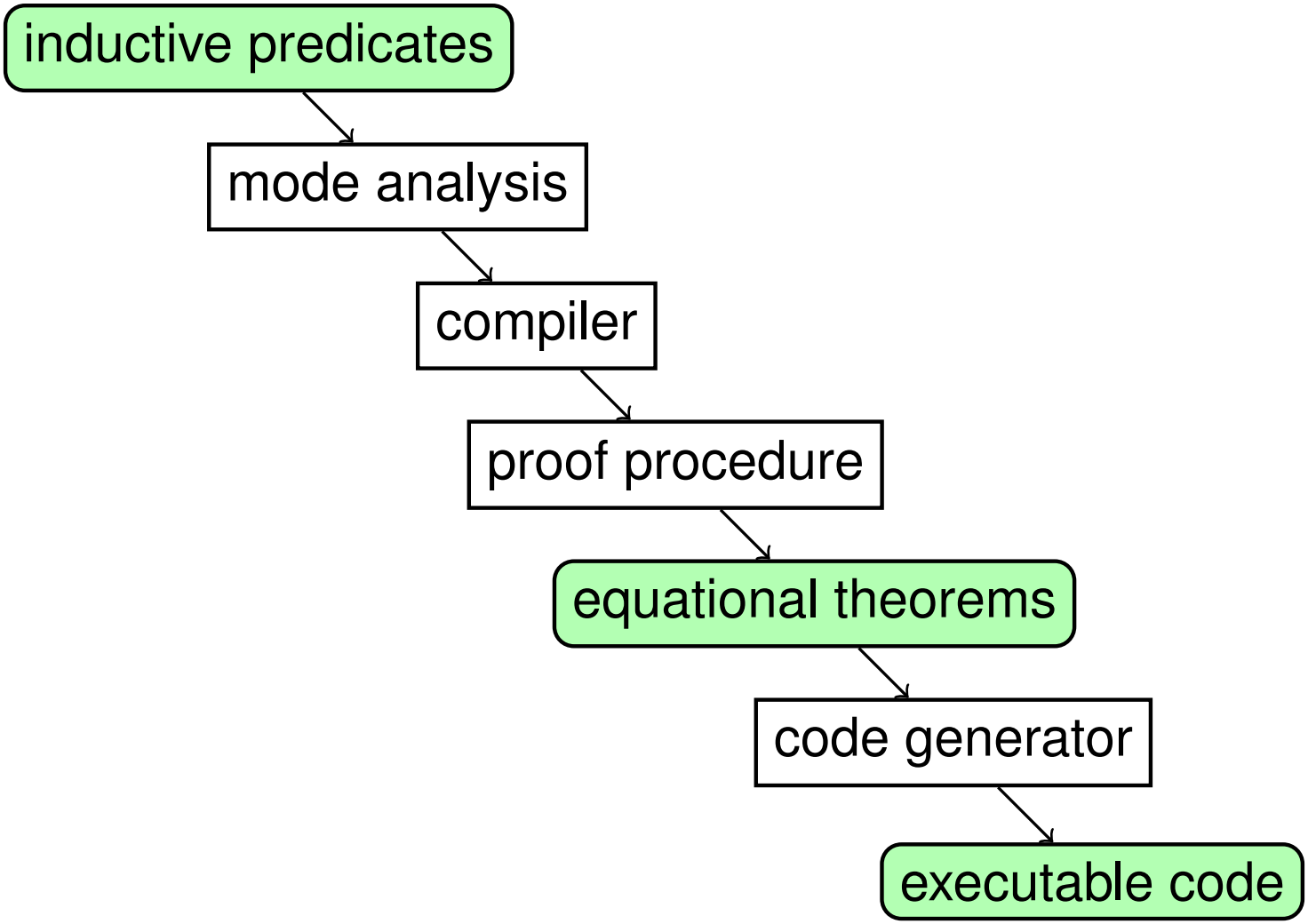
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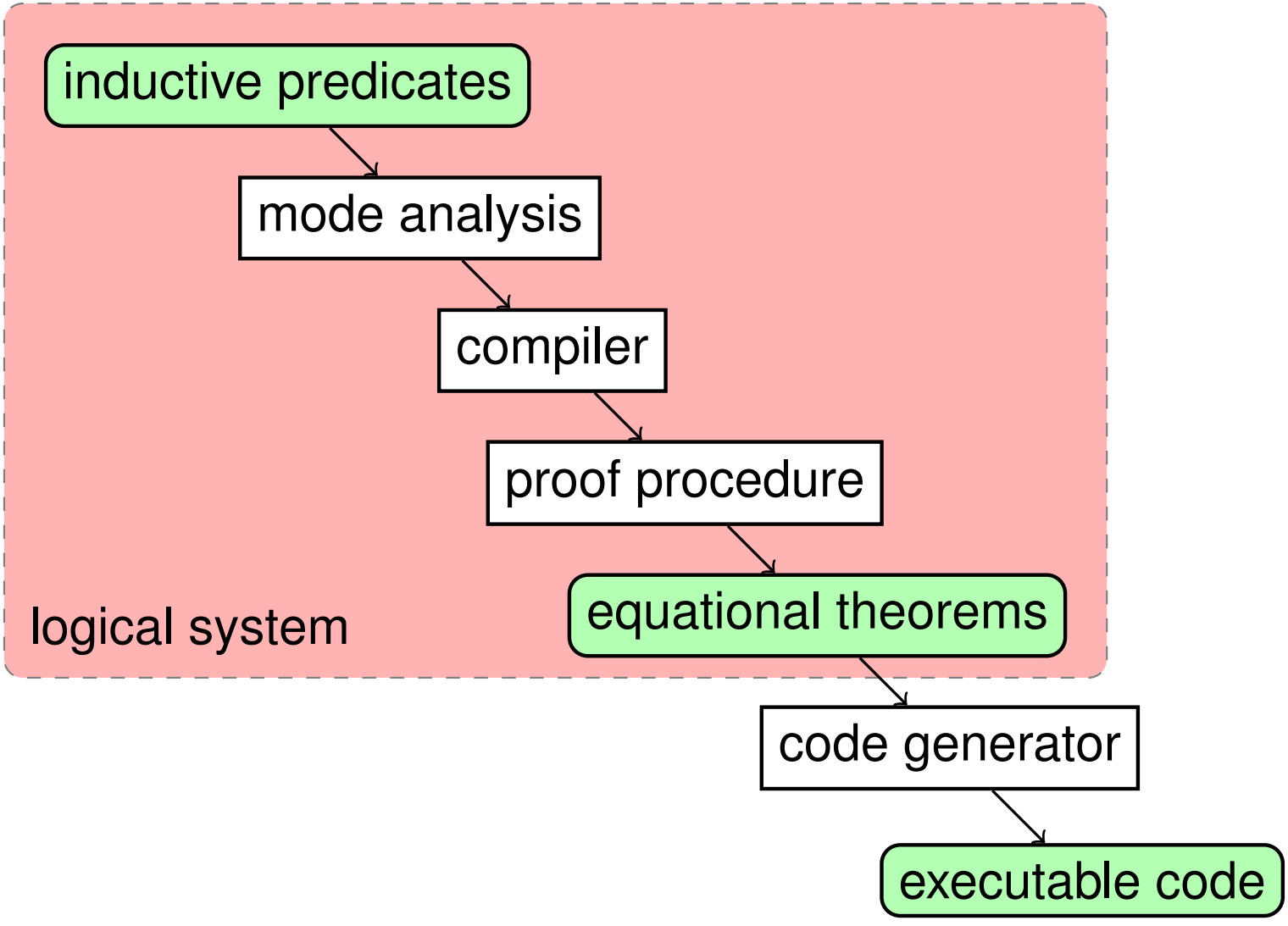
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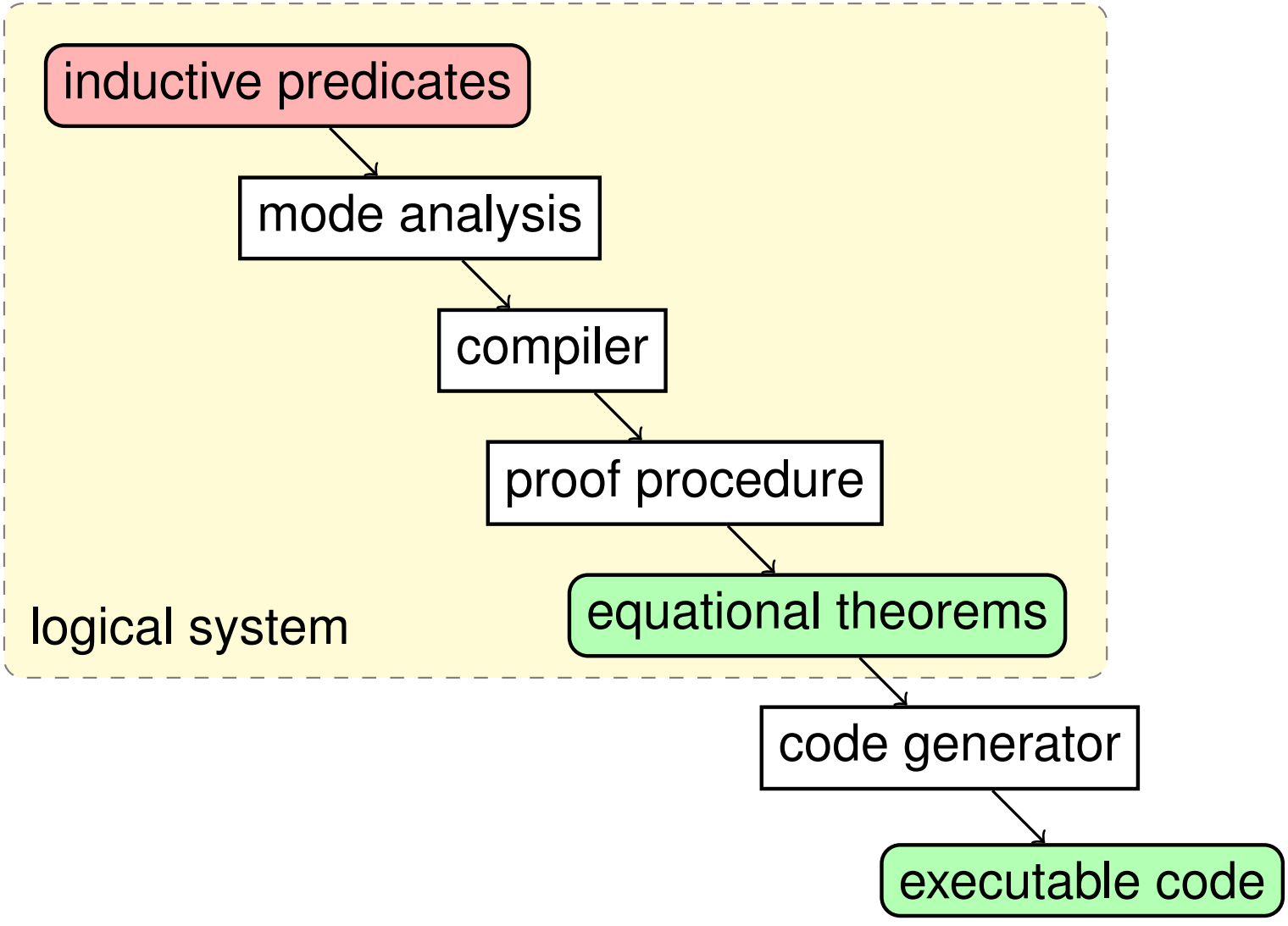
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Inductive Predicates

Example: append

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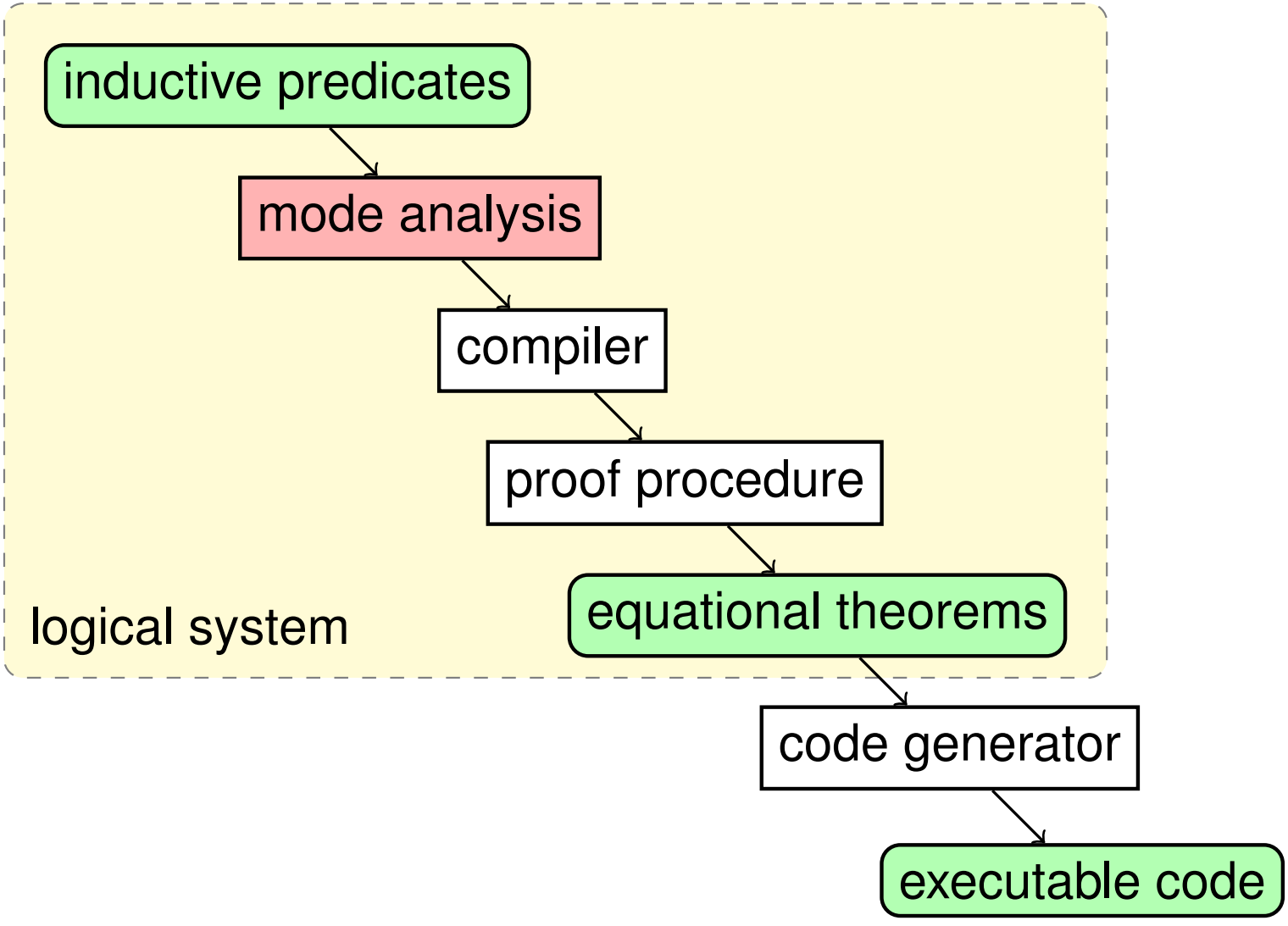
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Mode Analysis

A mode requires a dataflow of ground values in the introduction rules.

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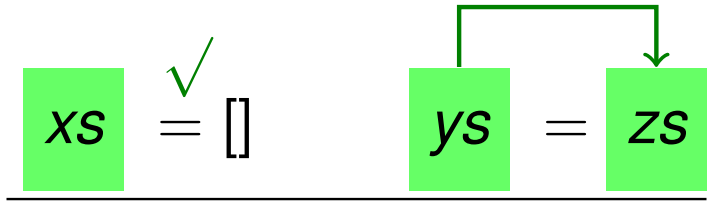

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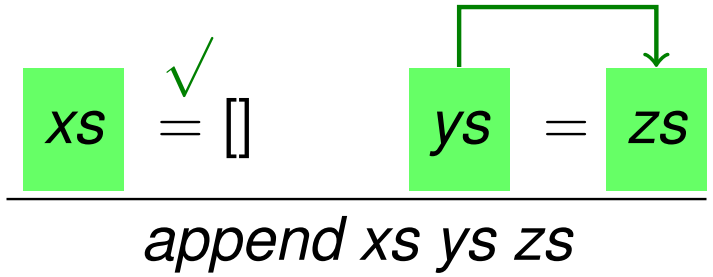

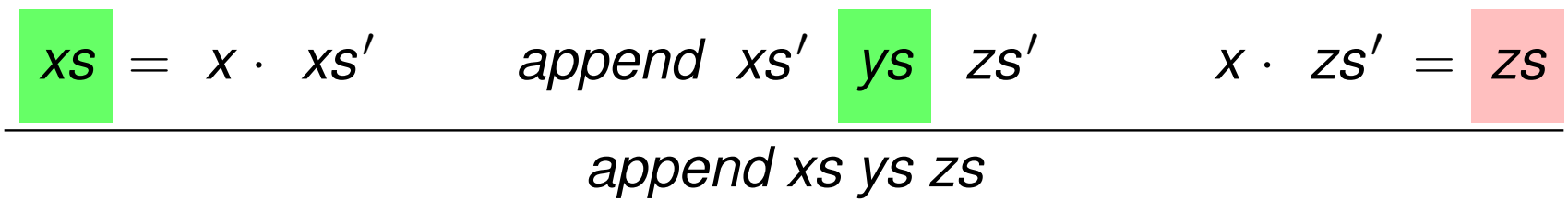

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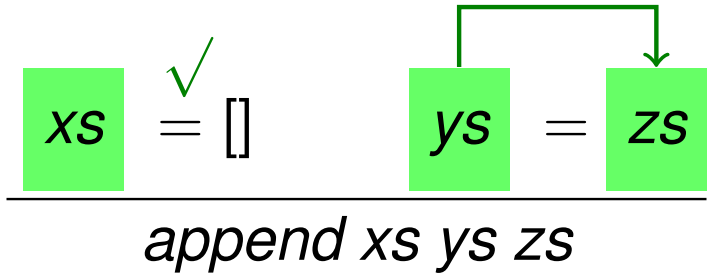

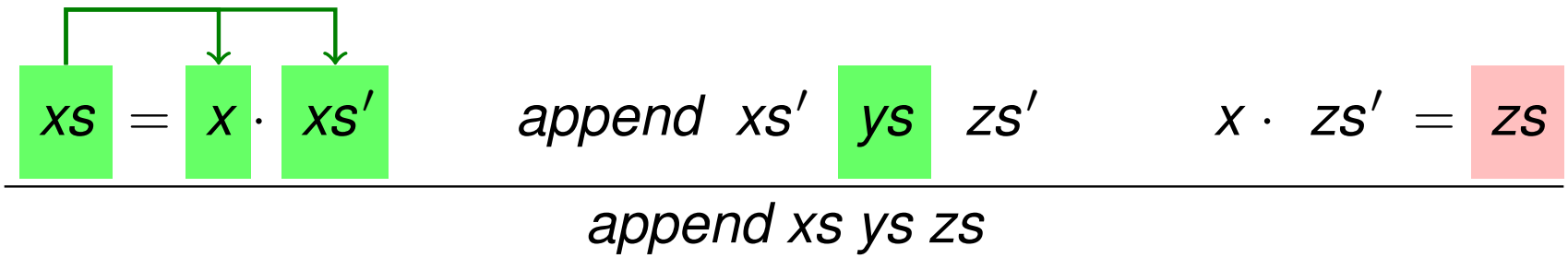

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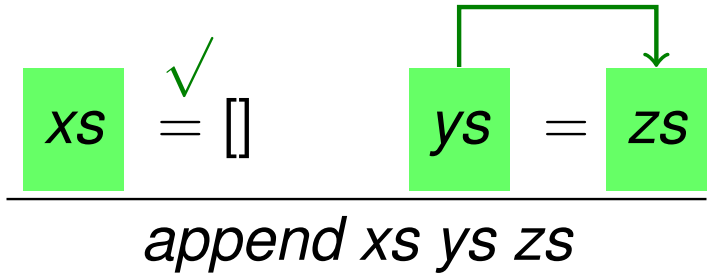

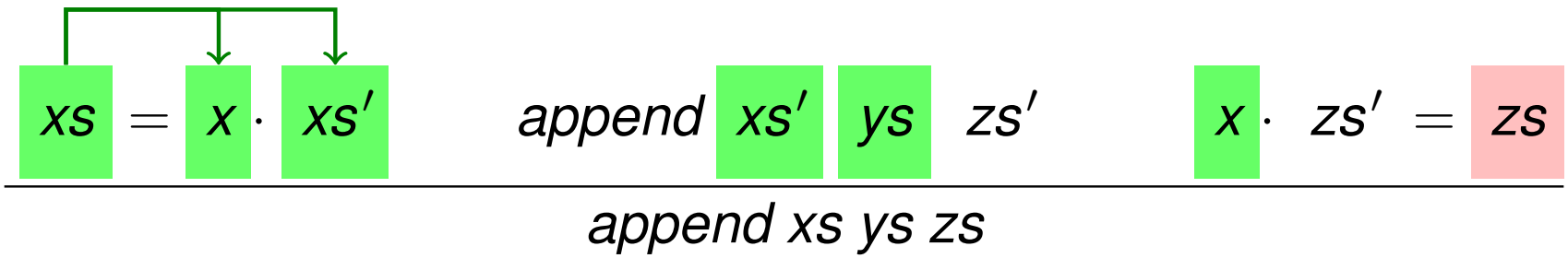

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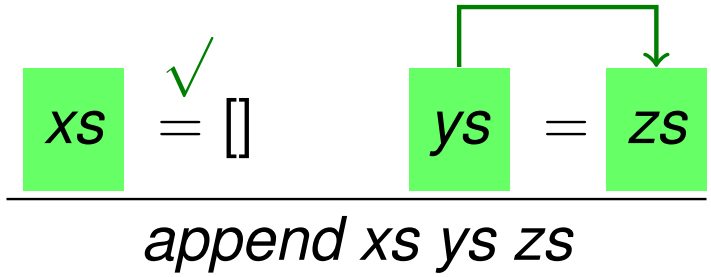

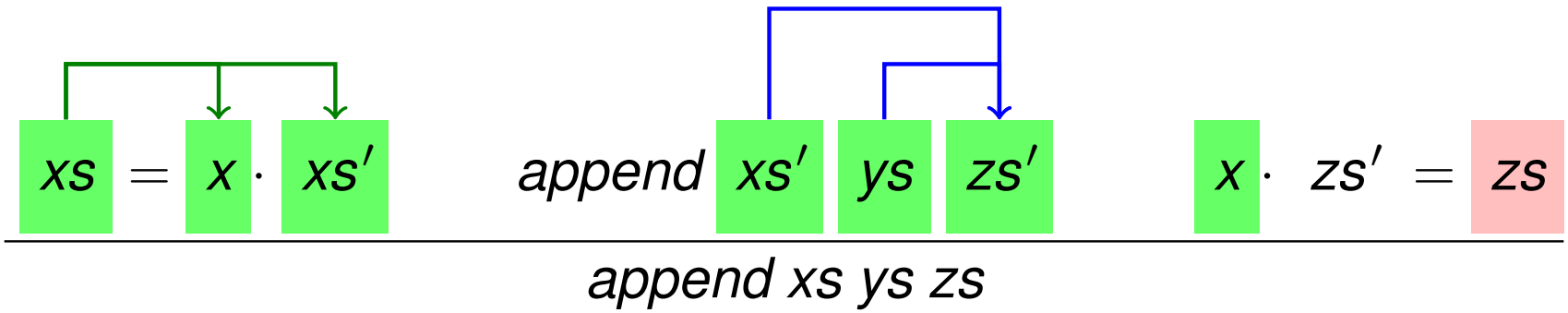

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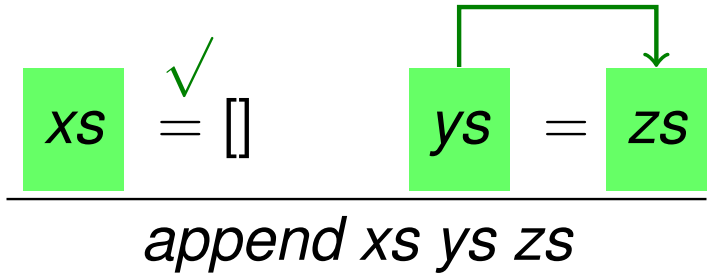

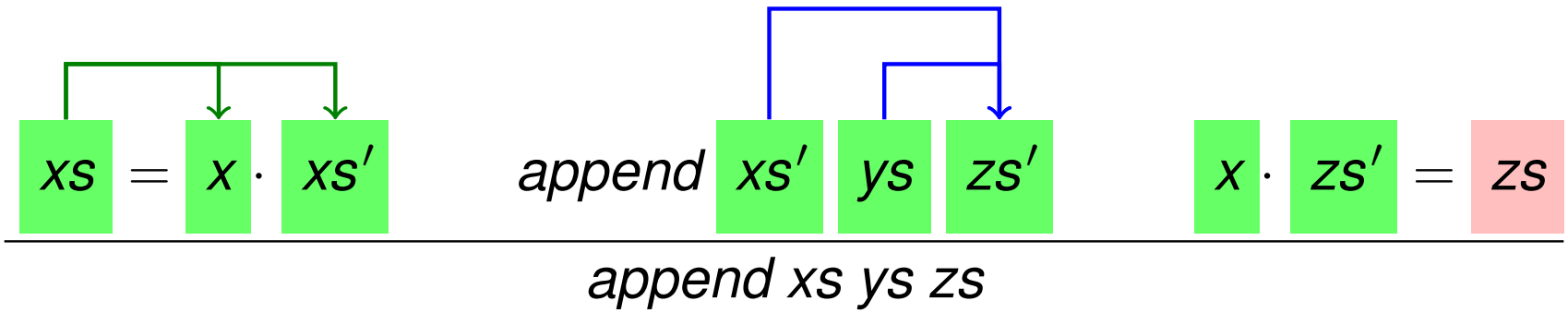

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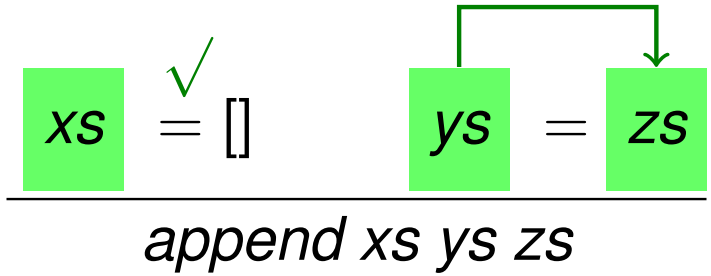

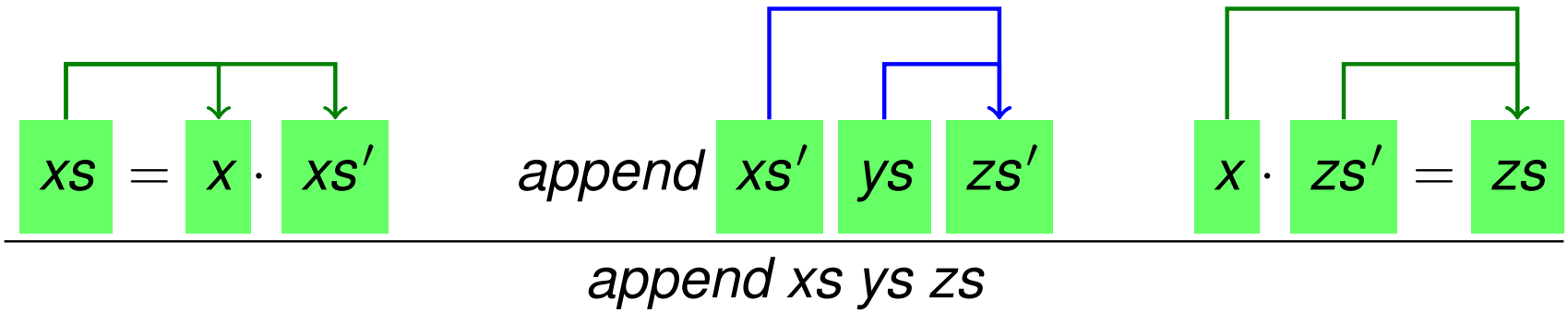

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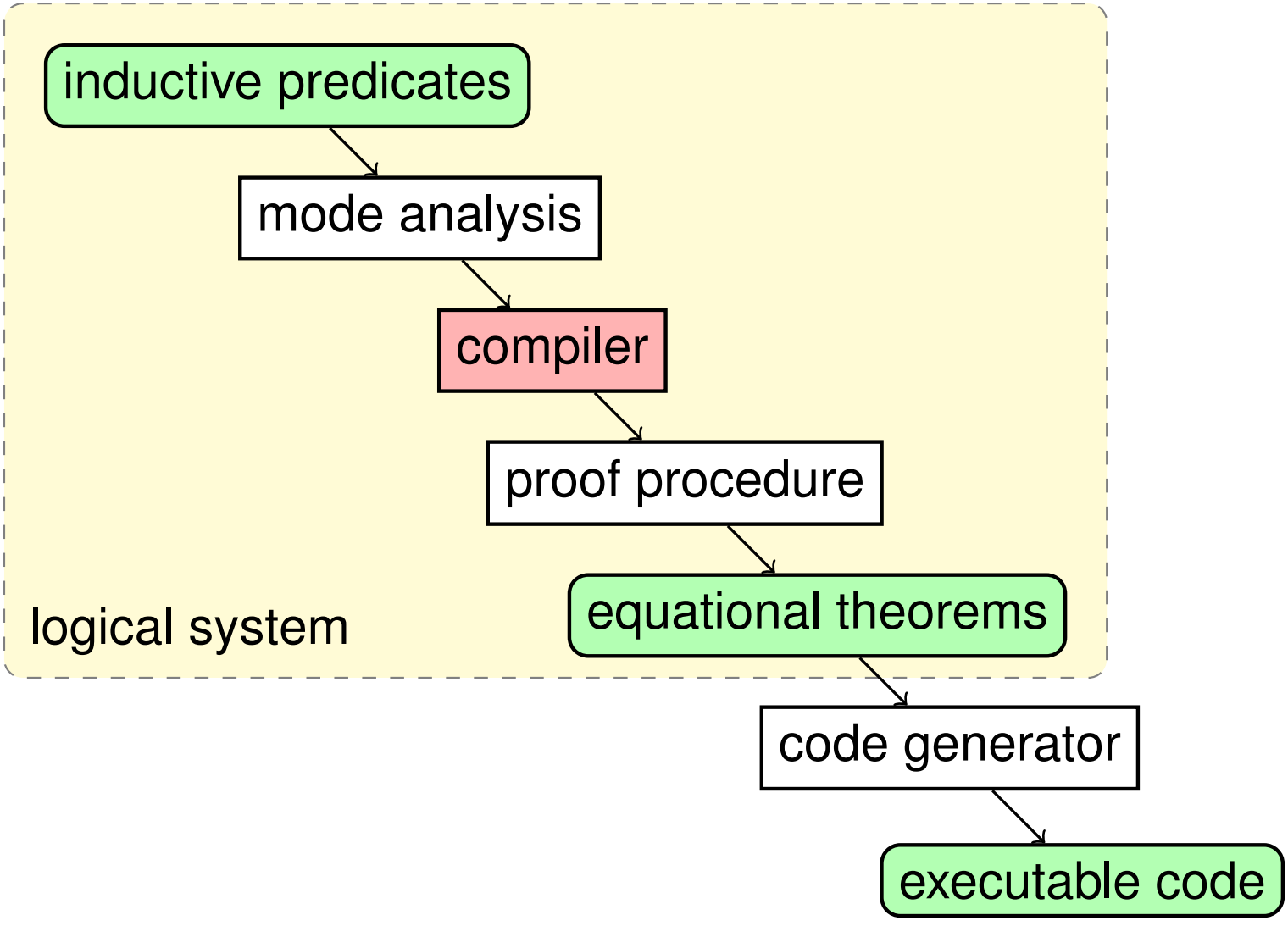
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Compilation of code equations

Basic set operations

- empty set: $\emptyset$
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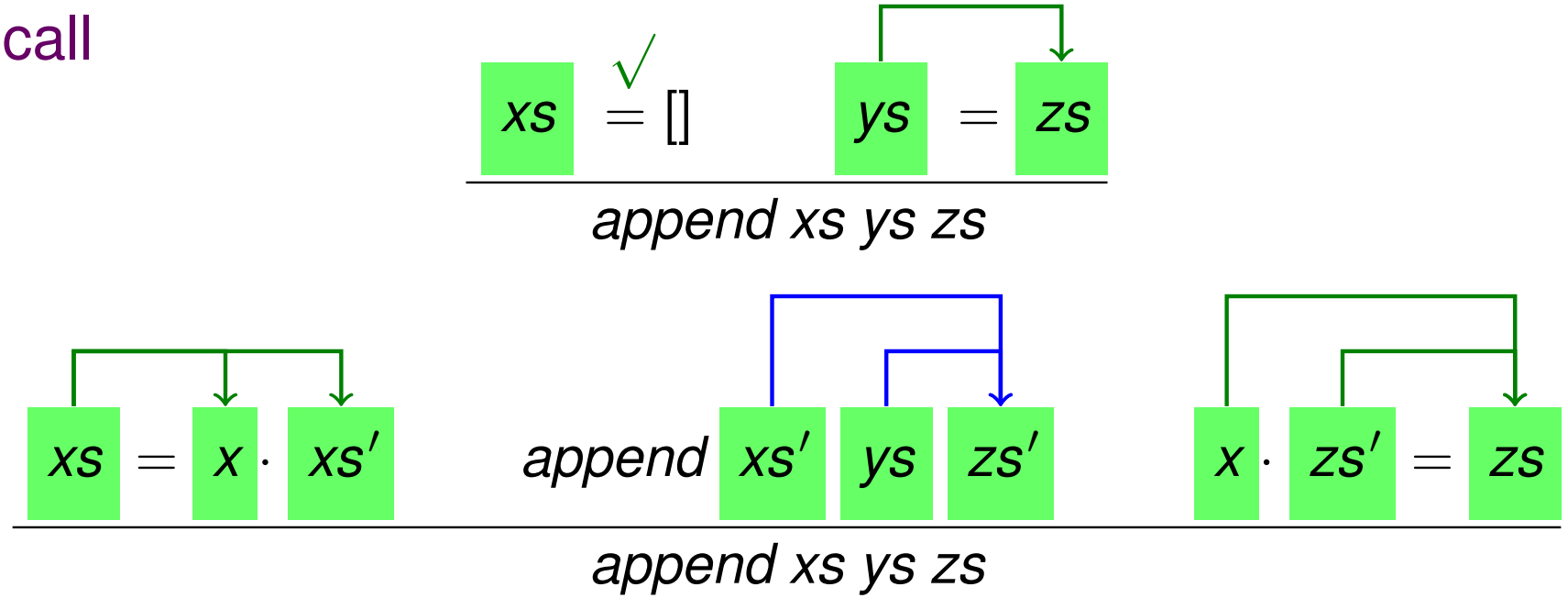
  $(\gg\!=) :: \alpha\ set \Rightarrow (\alpha \Rightarrow \beta\ set) \Rightarrow \beta\ set$
  applying function $f :: \alpha \Rightarrow \beta\ set$
  to all elements of $S :: \alpha\ set$ and merging the results, i.e. $\bigcup f\ `\ S$

Compilation of code equations

Recall



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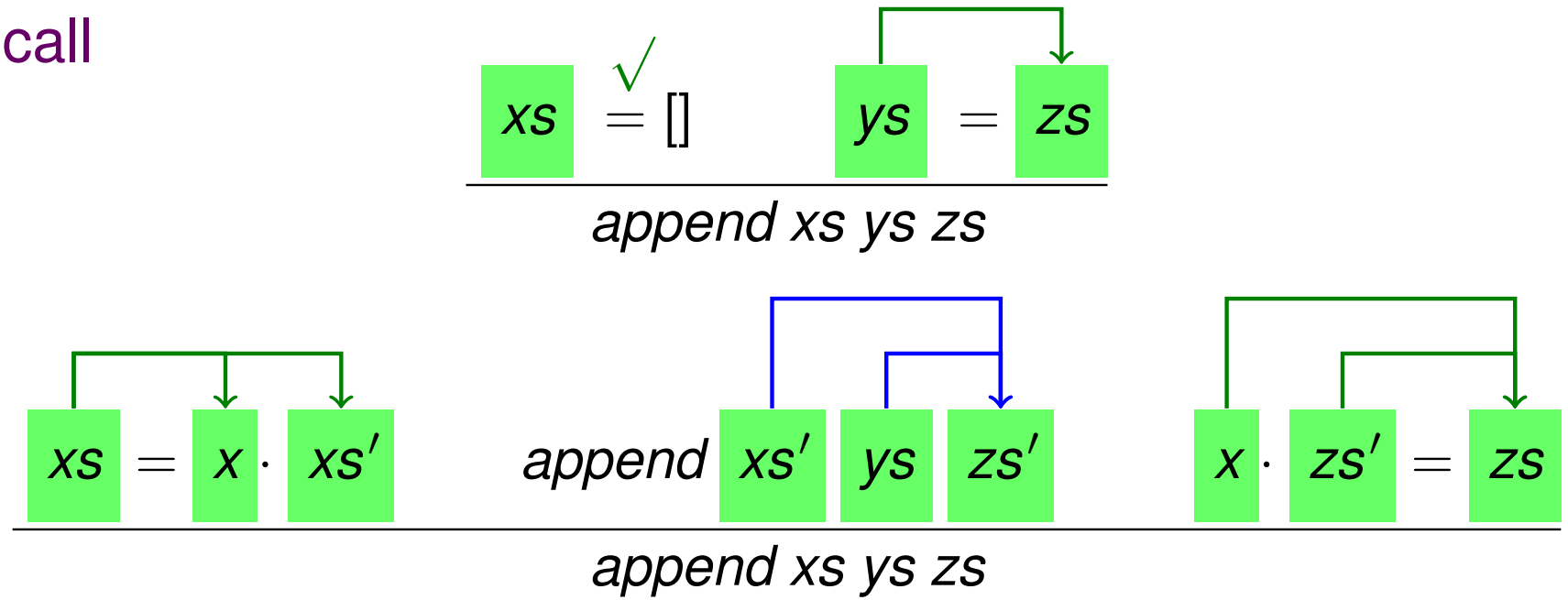

Code equation of *append*

$$append_{\{1,\,2\}}\ xs\ ys =$$
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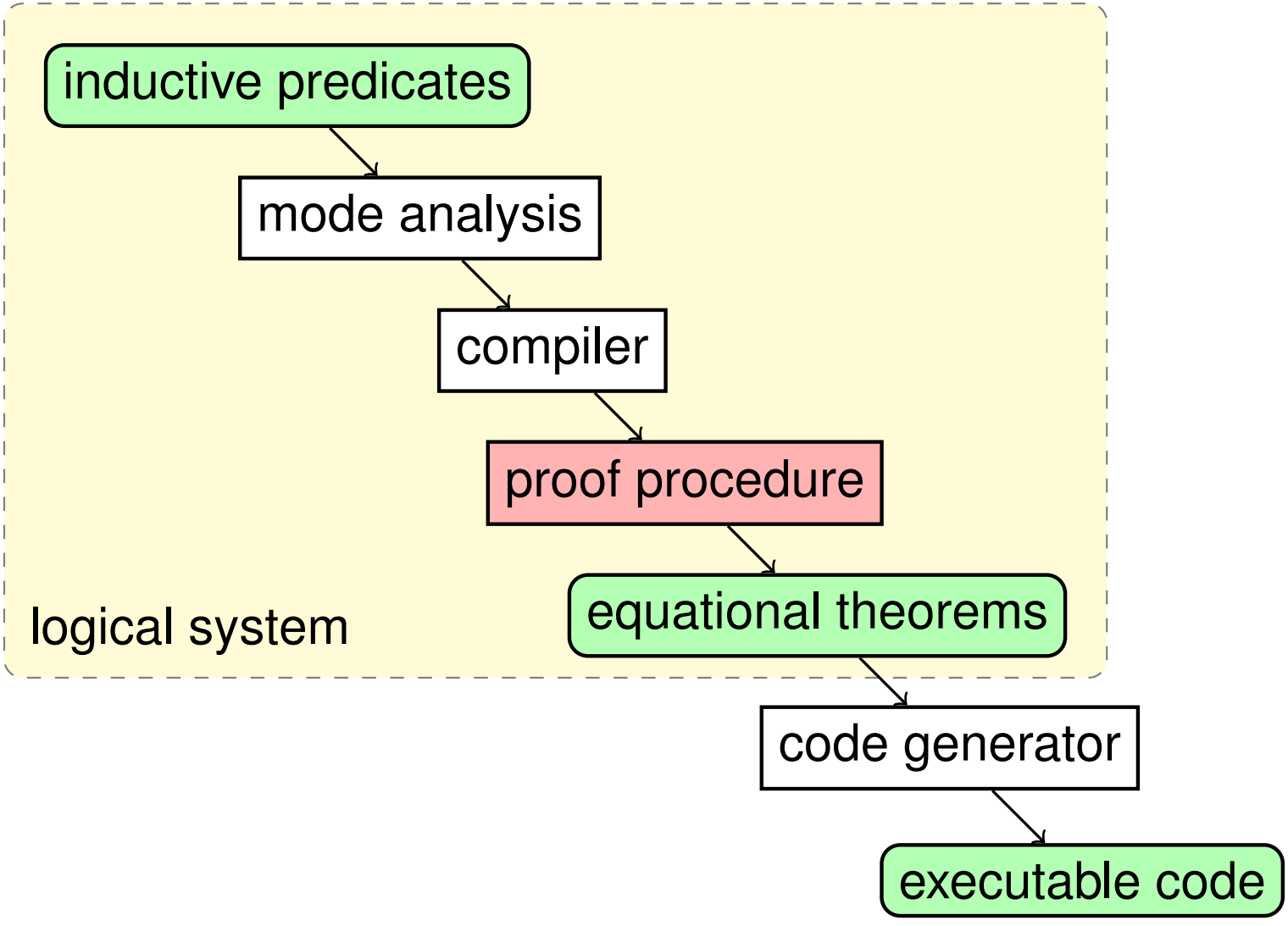
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- No interdependence of mutually recursive functions

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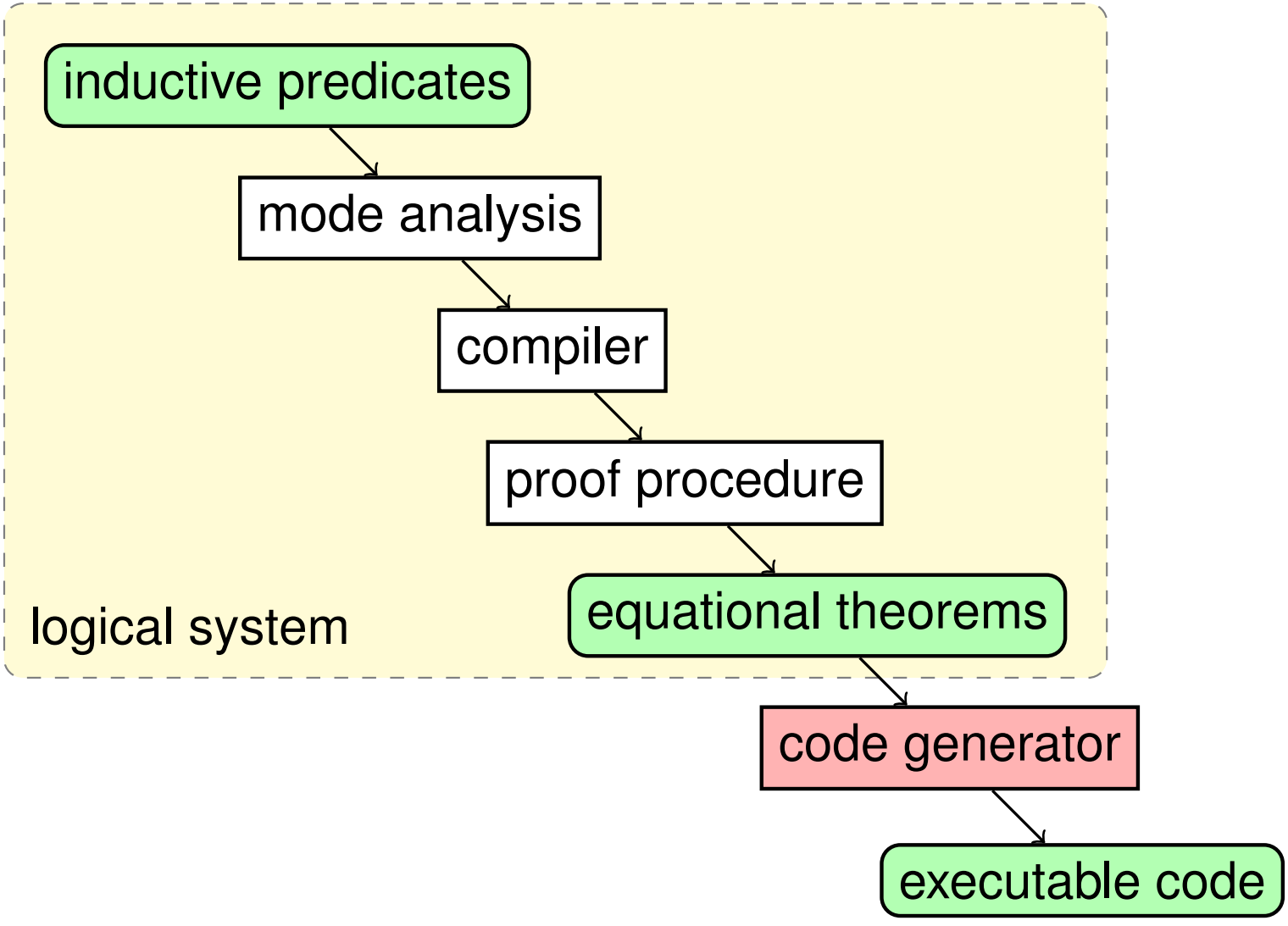
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- lazy evaluation in eager language, i.e. use explicit closures to delay computation

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datatype α *seq* $=$ *Empty* | *Insert* α (α *set*) | *Union* (α *set list*)

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Empty $\Rightarrow$ $\emptyset$ | *Insert x xq* $\Rightarrow$ $\{x\} \cup$ *xq* | *Union xqs* $\Rightarrow$ $\bigcup$ *xqs*)

Mapping to Standard ML

Empty :: α *seq*
Insert :: $\alpha \Rightarrow \alpha$ *set* $\Rightarrow \alpha$ *seq*
Union :: α *set list* $\Rightarrow \alpha$ *seq*
Seq :: (*unit* $\Rightarrow$ α *seq*) $\Rightarrow$ α *set*

Lazy-Evaluation Model of Sets

Representation for Sets

datatype α *seq* = *Empty* | *Insert* α (α *set*) | *Union* (α *set list*)

Seq :: (*unit* $\Rightarrow$ α *seq*) $\Rightarrow$ α *set*
Seq f = (*case f* () *of*
Empty $\Rightarrow$ $\emptyset$ | *Insert x xq* $\Rightarrow$ $\{x\} \cup$ *xq* | *Union xqs* $\Rightarrow$ $\bigcup$ *xqs*)

Mapping to Standard ML

Empty :: α *seq*
Insert :: $\alpha \Rightarrow \alpha$ *set* $\Rightarrow \alpha$ *seq*
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Seq :: (*unit* $\Rightarrow \alpha$ *seq*) $\Rightarrow \alpha$ *set*

are chosen to be uninterpreted in SML:

```
datatype 'a seq = Empty | Insert of 'a * 'a set | Union of 'a set list
and 'a set = Seq of (unit -> 'a seq);
```

Executable Equations of Set Operations

We derive executable equations for the four set operations:

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$$\begin{aligned}
&\mathit{Seq}\ g \gg\!= f = \\
&\quad \mathit{Seq}\ (\lambda u.\ \mathit{case}\ g\ ()\ \mathit{of}\ \mathit{Empty} \Rightarrow \mathit{Empty} \\
&\qquad\qquad \mid \mathit{Insert}\ x\ xq \Rightarrow \mathit{Union}\ [f\ x,\ xq \gg\!= f] \\
&\qquad\qquad \mid \mathit{Union}\ xqs \Rightarrow \mathit{Union}\ (\mathit{map}\ (\lambda x.\ x \gg\!= f)\ xqs))
\end{aligned}$$

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&\qquad\qquad \mid \mathit{Union}\ xqs \Rightarrow \mathit{Union}\ (\mathit{map}\ (\lambda x.\ x \gg\!= f)\ xqs))
\end{aligned}$$

$$\begin{aligned}
&\mathit{Seq}\ f \cup \mathit{Seq}\ g = \\
&\quad \mathit{Seq}\ (\lambda u.\ \mathit{case}\ f\ ()\ \mathit{of}\ \mathit{Empty} \Rightarrow g\ () \\
&\qquad\qquad \mid \mathit{Insert}\ x\ xq \Rightarrow \mathit{Insert}\ x\ (xq \cup \mathit{Seq}\ g) \\
&\qquad\qquad \mid \mathit{Union}\ xqs \Rightarrow \mathit{Union}\ (xqs\ @\ [\mathit{Seq}\ g]))
\end{aligned}$$

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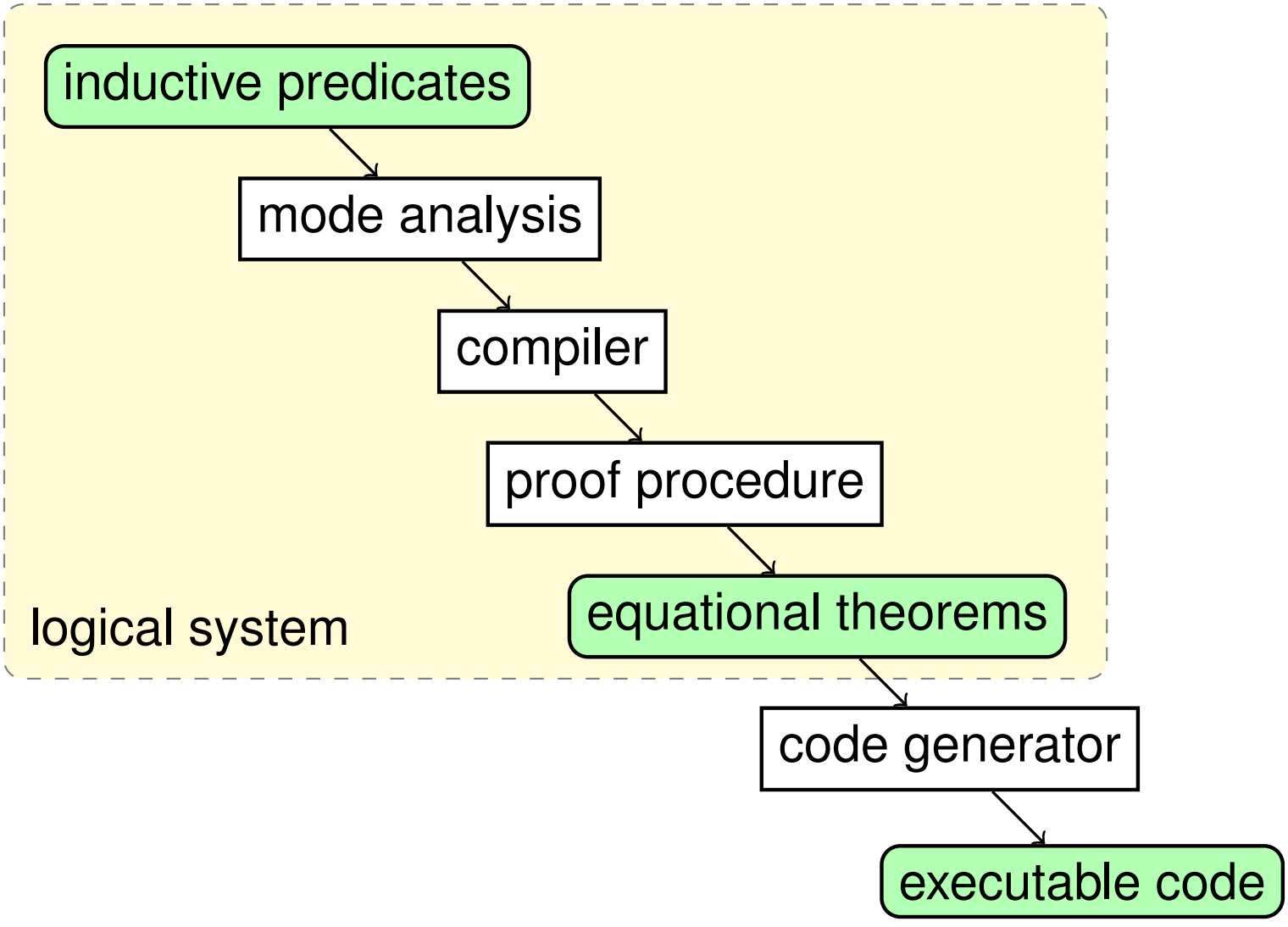
&\qquad\qquad \mid \mathit{Union}\ xqs \Rightarrow \mathit{Union}\ (xqs\ @\ [\mathit{Seq}\ g]))
\end{aligned}$$

Note: All equations are guarded by $\mathit{Seq}\ (\lambda u.\ \ldots)$!

Overview


inductive predicates
mode analysis
compiler
proof procedure
equational theorems
logical system
code generator
executable code