Solving Geometric Construction Problems

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Agenda

- Motivation and Our Goals
- Related Work
- Problem Description and Our Approach
- Results
- Future Work

Motivation and Our Goals

- Automating geometry constructions is an important, but a hard task

- Not many successful approaches

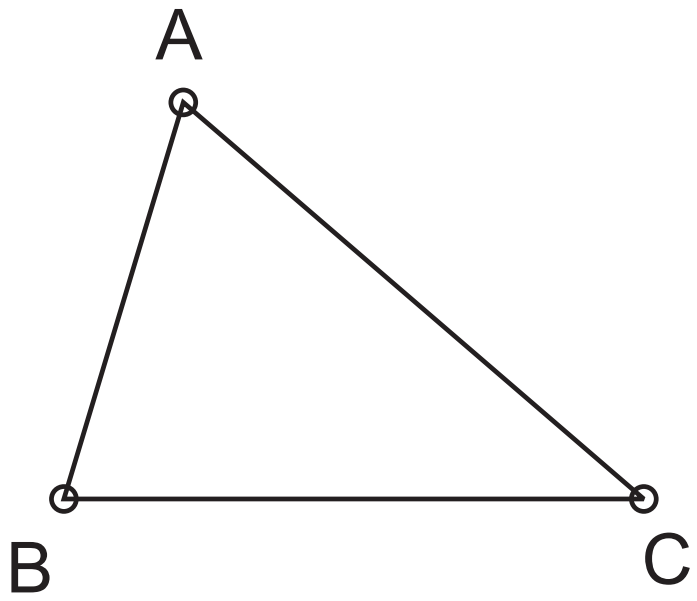
- Short-term goal – automatic generation of analysis for construction problems of modest hardness

- Long-term goal – generation of construction, along with proof of correctness and discussion of number of solutions

Related Work

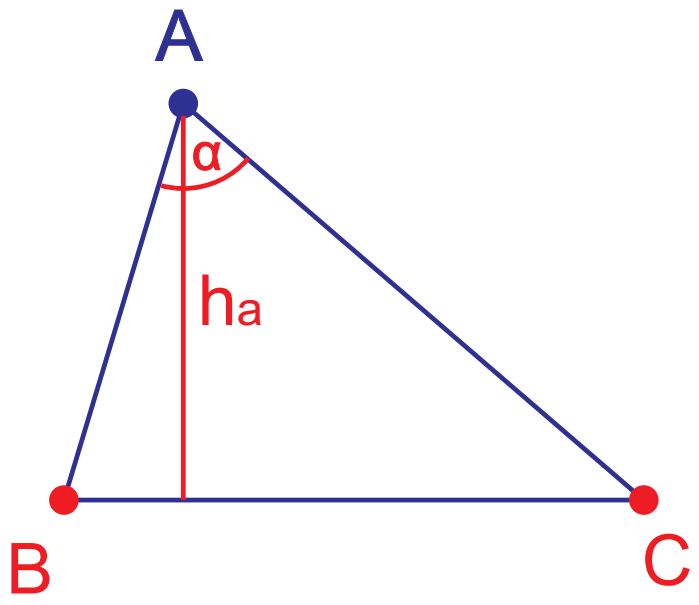
- GRAMY (Matsuda/Vanlehn, 2004)
- Axiomatization of geometries of ruler and compass (Pambuccian, 2008)
- Formalization of axiomatic system for ruler and compass Euclidean geometry (Duprat, 2008)
- Automating geometry constructions as a program synthesis problem (Gulwani, 2010)

Problem Description



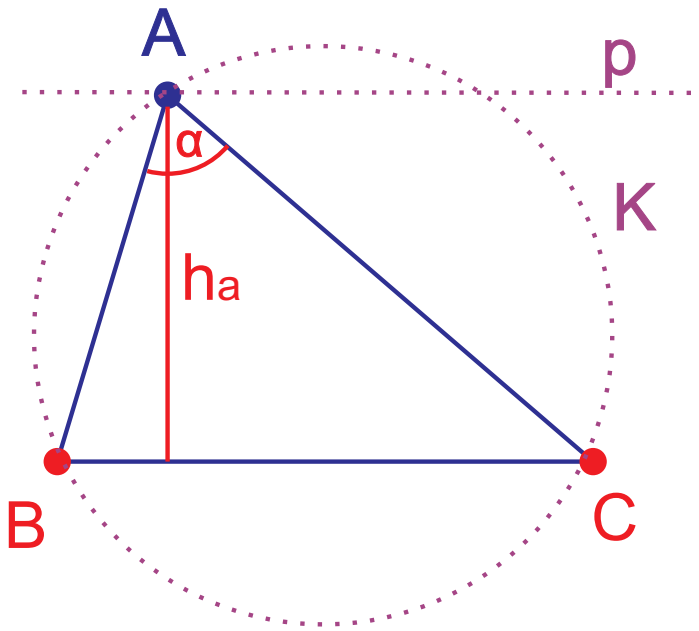
General problem: to construct a triangle ABC given some of its elements

Problem Description



Specific problem: to construct a triangle ABC given two vertices: B and C, the length of the altitude h_a and the size of the angle α

Problem Description



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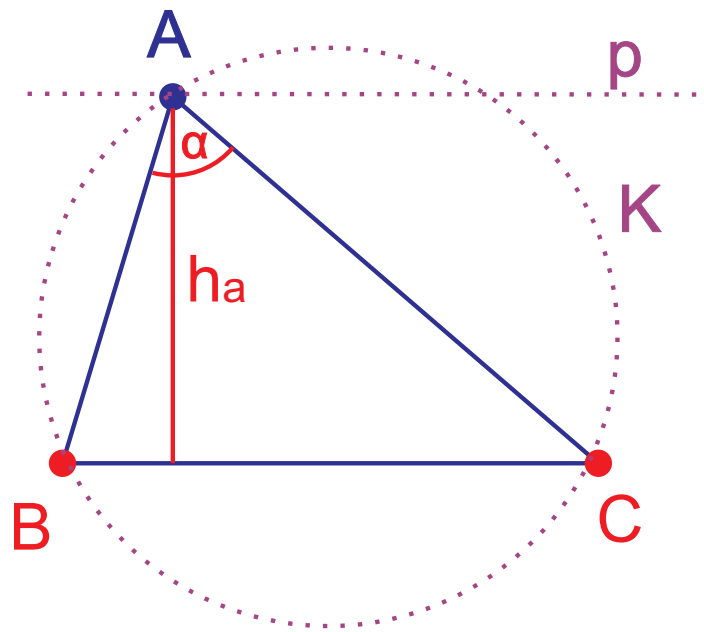

Analysis

Construction

Proof

Discusion

Our approach - algorithm and implementation

- Objects are assigned status: known/sought
- Combination of forward and backward chaining
- Two types of rules
  - Forward rules
  - Backward rules
- Control of recursion depth
- Prolog program

Example of forward and backward rule

```
FW_rule: circumcirle(A,B,C) is known if:
           circumcircle(A,B,C) is potentially sought
            and point B is known
            and point C is known
            and angle(A,B,C) is known


BW_rule: circumcircle(A,B,C) is potentially sought if:
           point A is potentially sought
            or
           point B is potentially sought
            or
           point C is potentially sought
```

Example

Input:

```
obj(1,point,b,known,_).
obj(1,point,c,known,_).
obj(1,point,a,sought,_).
obj(1,length,distance(a,b,c),known,_).
obj(1,size,angle(a,b,c),known,_).
```

Query:

```
obj_search(1,point,a,known,‘proof1.txt‘).
```

Example

Output:

According to the rule $B1$, since point a is potentially sought, circumcircle of the triangle abc is potentially sought.

According to the rule $F2$, since points b and c are known and angle bac is known, circumcircle of the triangle abc is known.

According to the rule $B2$, since a is potentially sought, line through point a at given distance from line bc is potentially sought.

According to the rule $F1$, since points b and c are known and distance from point a to line bc is known, the line through point a at given distance from line bc is known.

According to the rule $F3$, since circumcircle of the triangle abc is known and a line through point a at given distance from line bc is known, point a is known.

Elapsed time: 0.906 seconds.

Preliminary results

- Tested on 15 simple problems
- Maximal depth of the proof is 3
- Maximal elapsed time is 45 seconds

Future (and current) work

- Expanding the corpus of construction problems (testing on some more difficult problems)
- Refining the set of rules
- Experimenting with fw-bw strategies
- Transition to resolution and Vampire prover
- Using Groebner bases - to get proofs and degenerate conditions