Incremental, Inductive Coverability

(to appear at CAV 2013)

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Max Planck Institute for Software Systems

March 30, 2013, Belgrade

Abstraction Using Petri Nets

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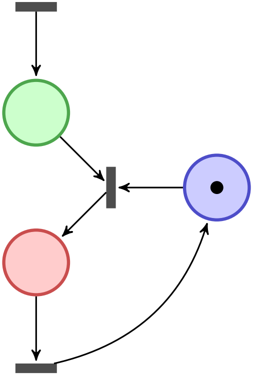

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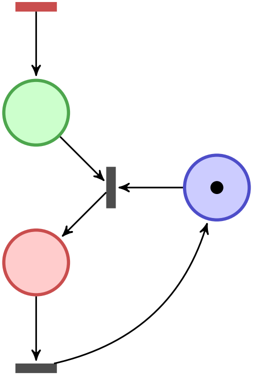

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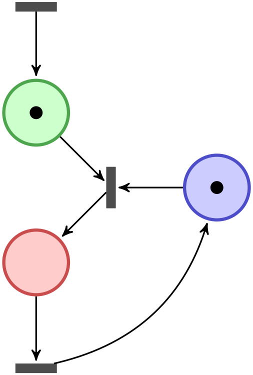

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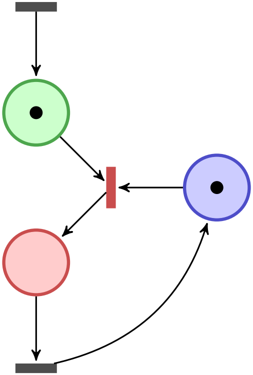

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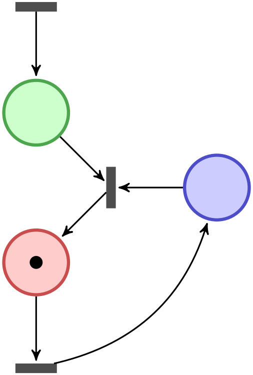

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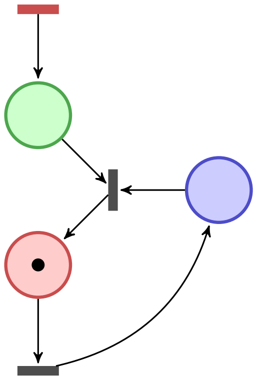

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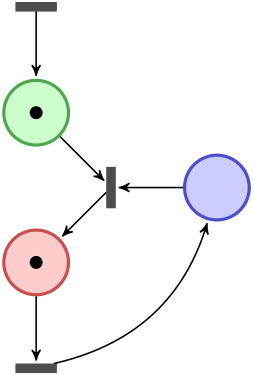

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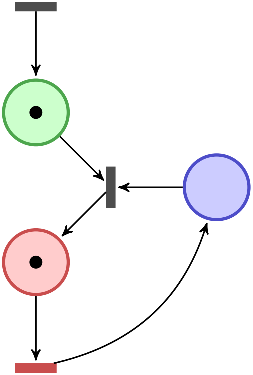

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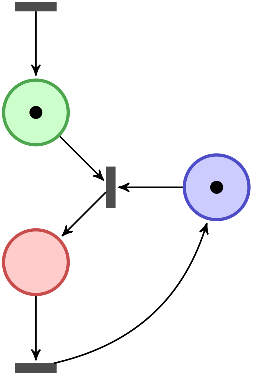

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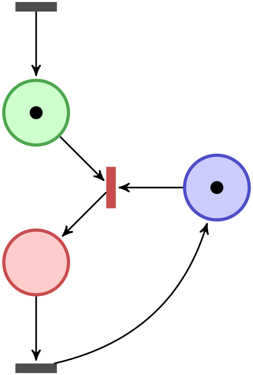

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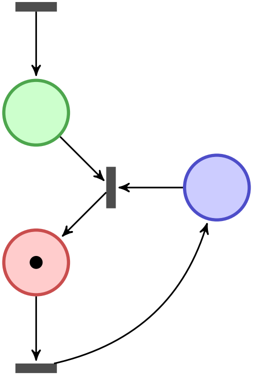

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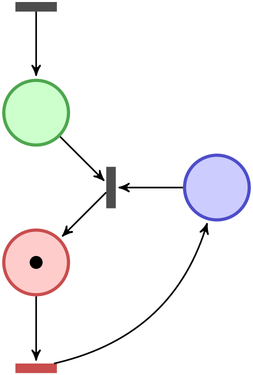

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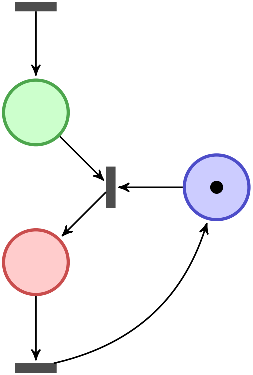

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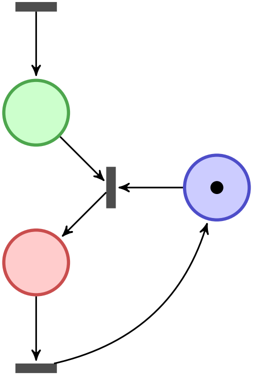
- Can two or more threads be in the critical section?
- Can we reach $(x, y, z) \geq (2, 0, 0)$?
- Can we cover $(2, 0, 0)$?



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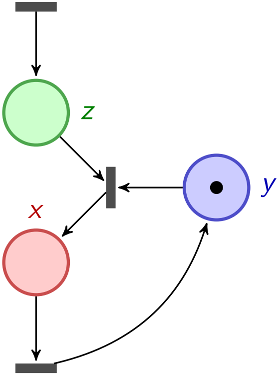

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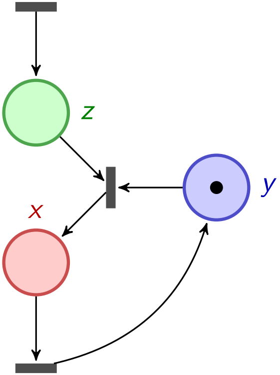

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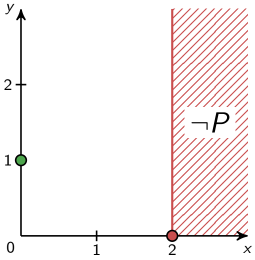

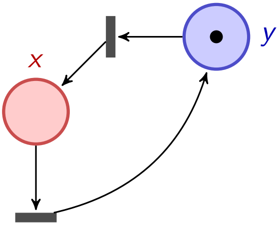

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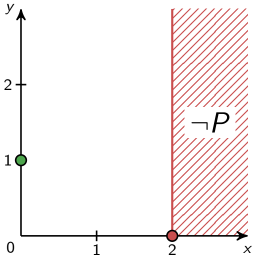

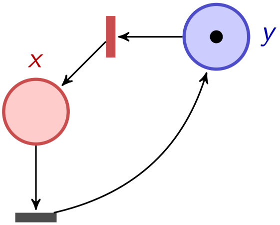

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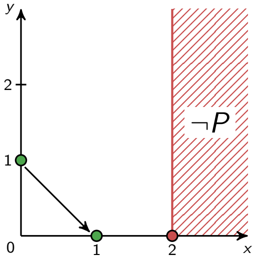

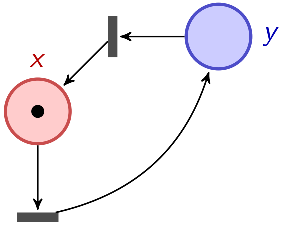

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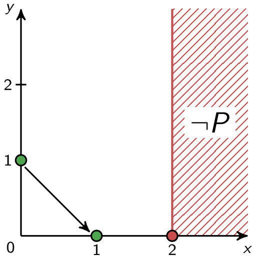

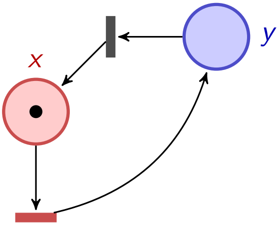

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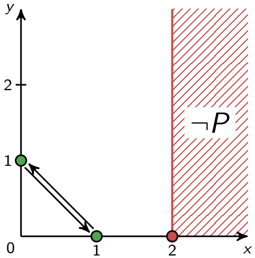

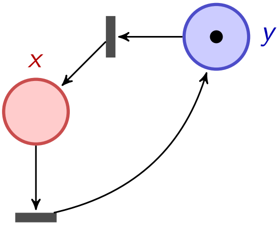

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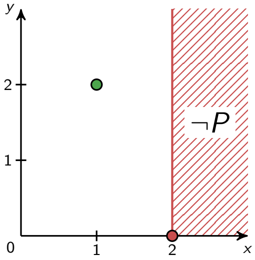

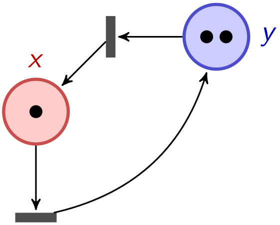

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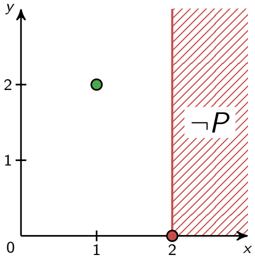

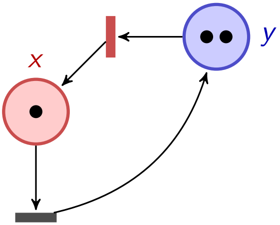

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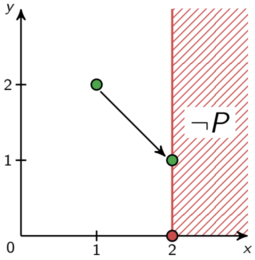

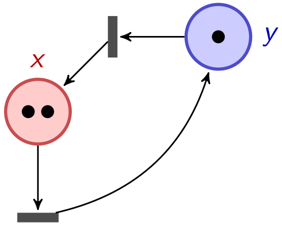

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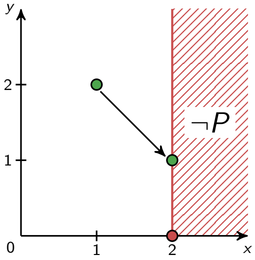

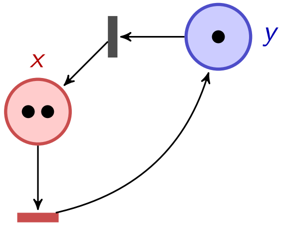

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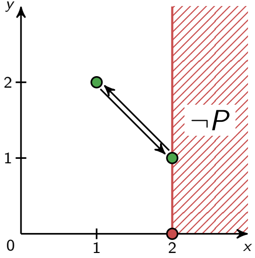

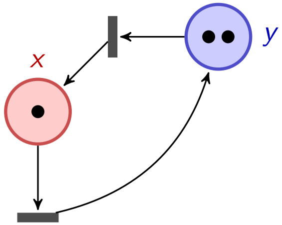

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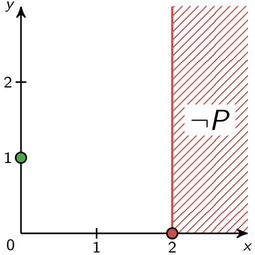

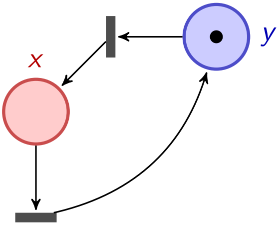

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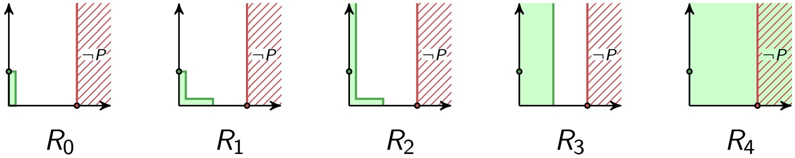

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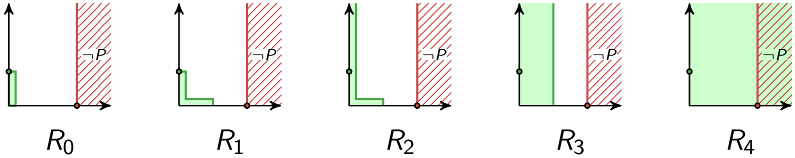

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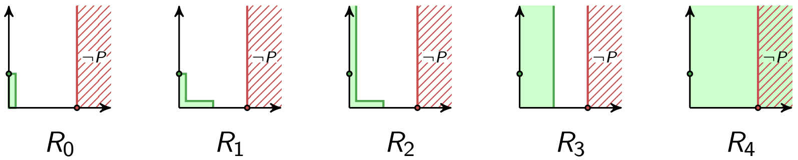

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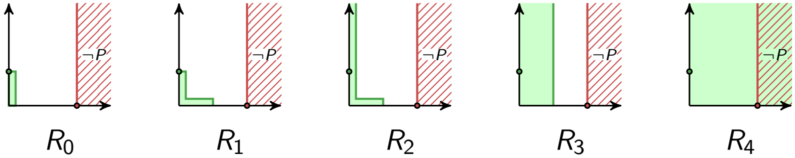

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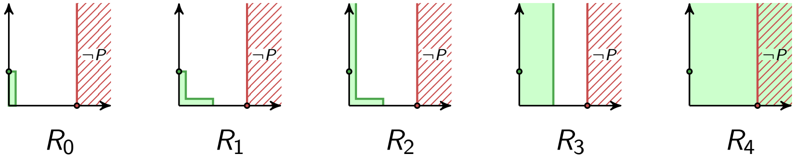

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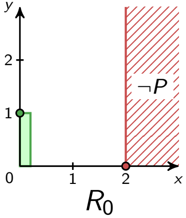

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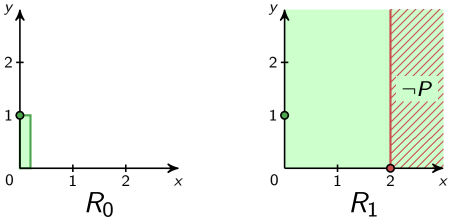

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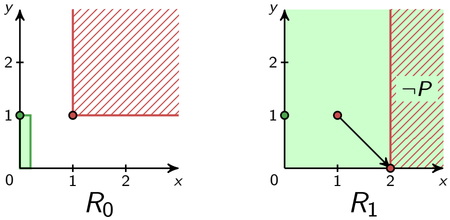

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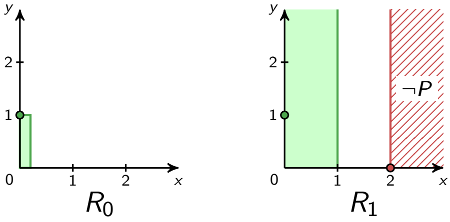

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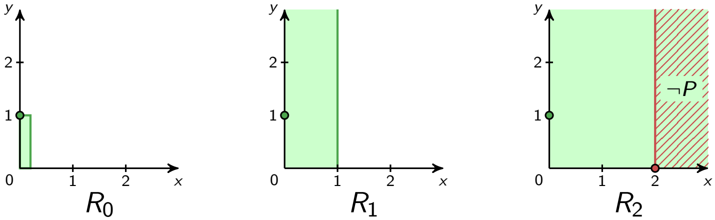

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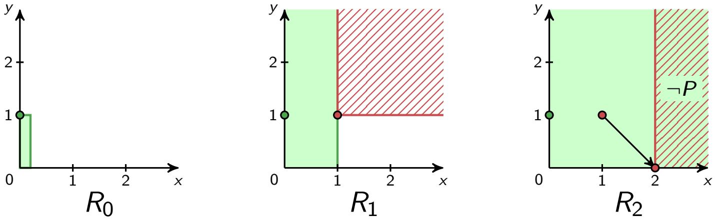

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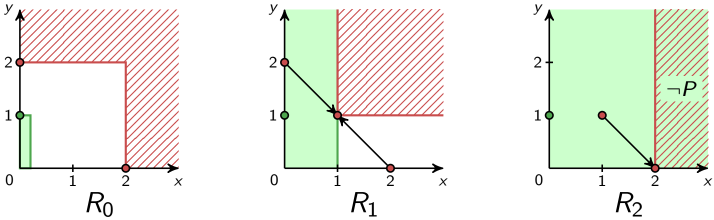

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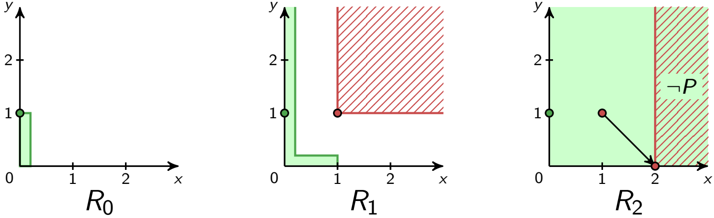

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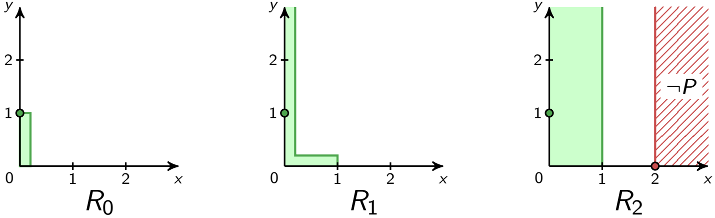

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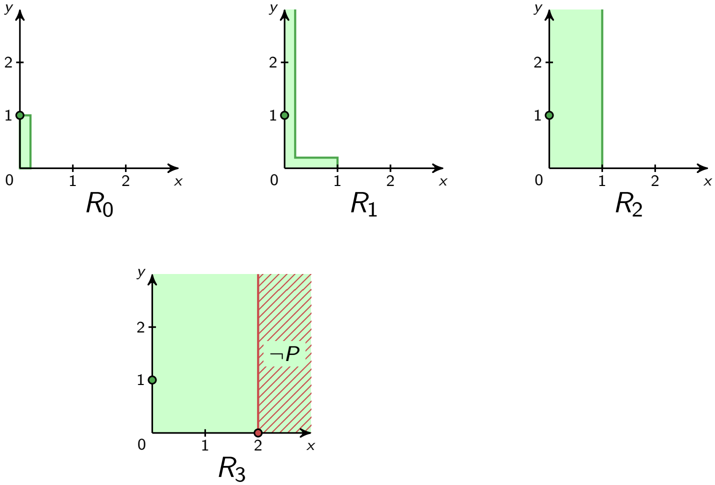

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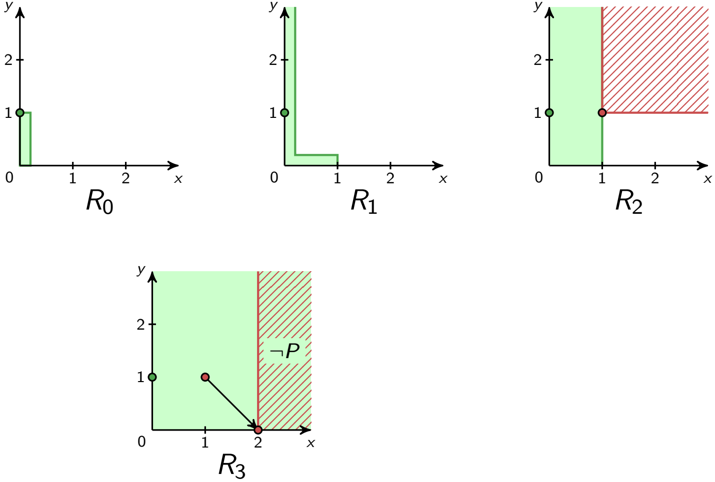

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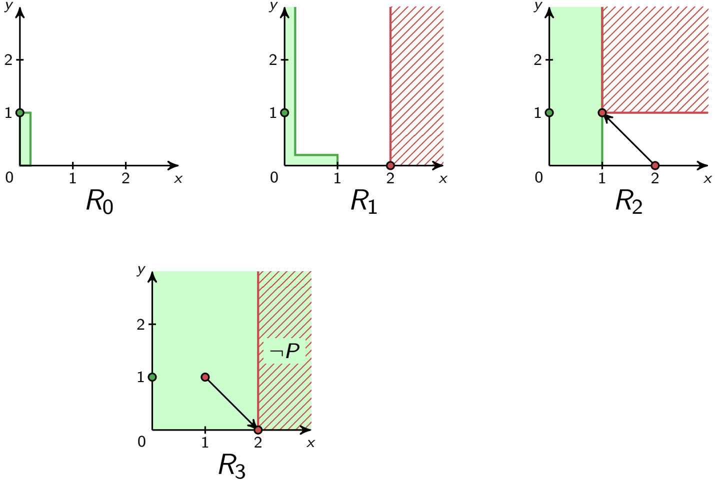

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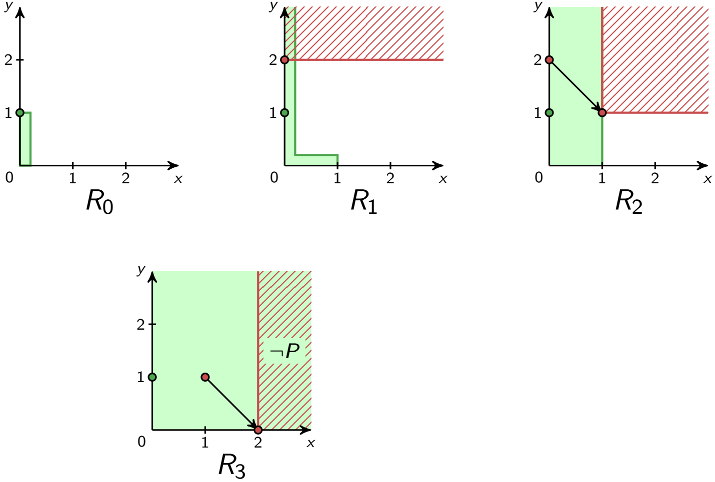

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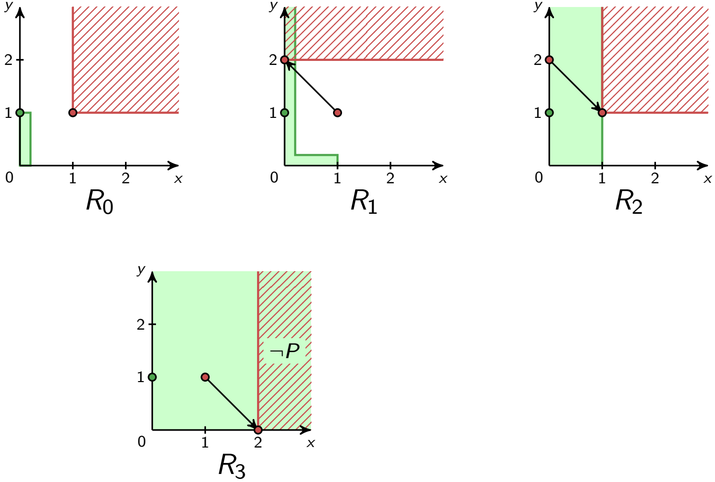

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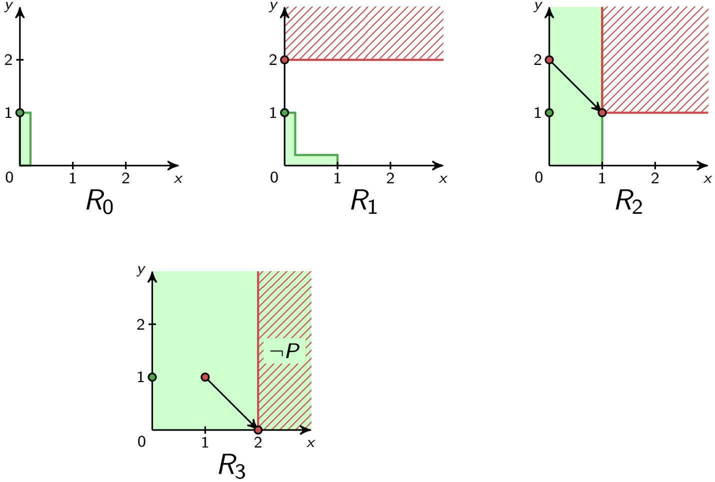

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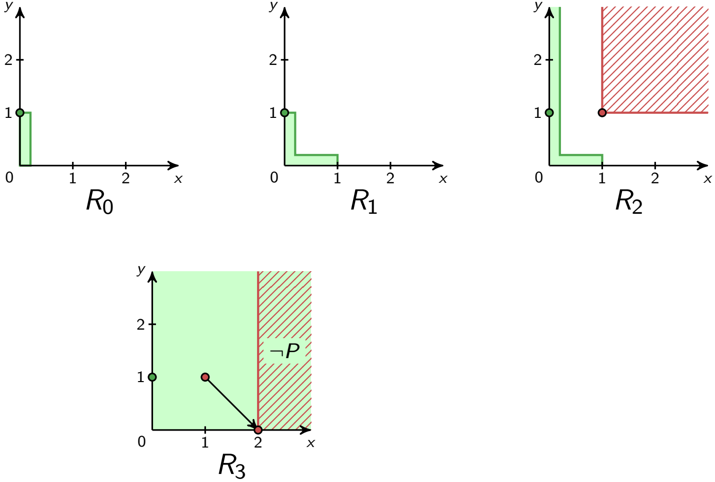

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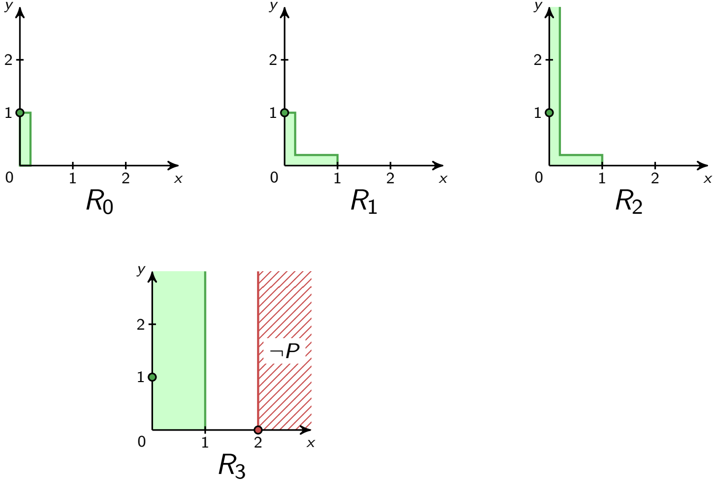

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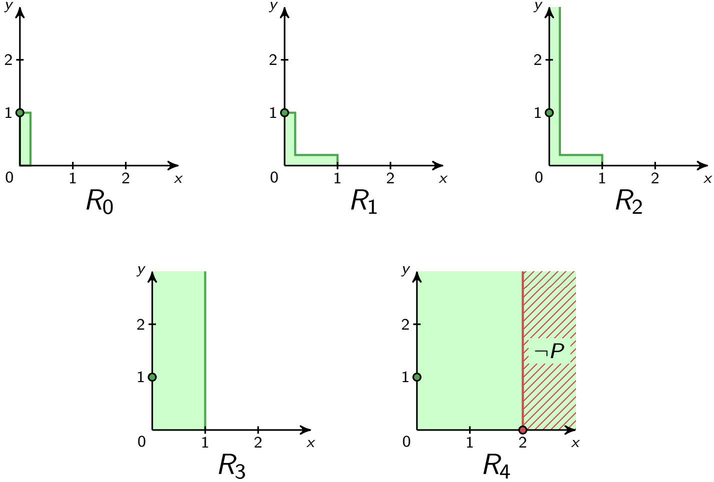

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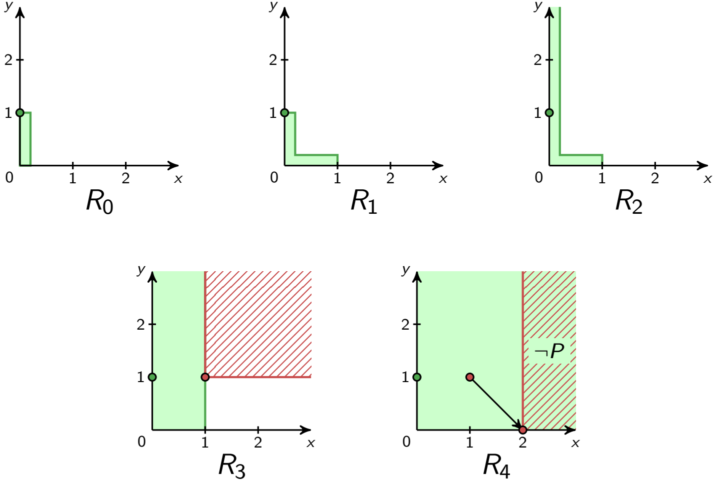

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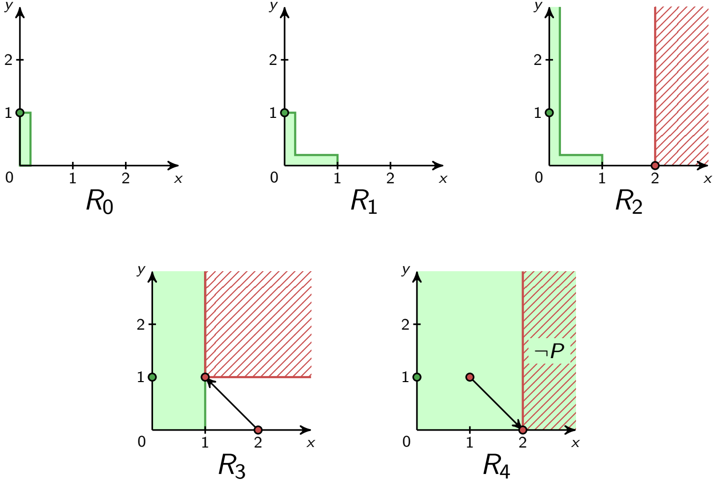

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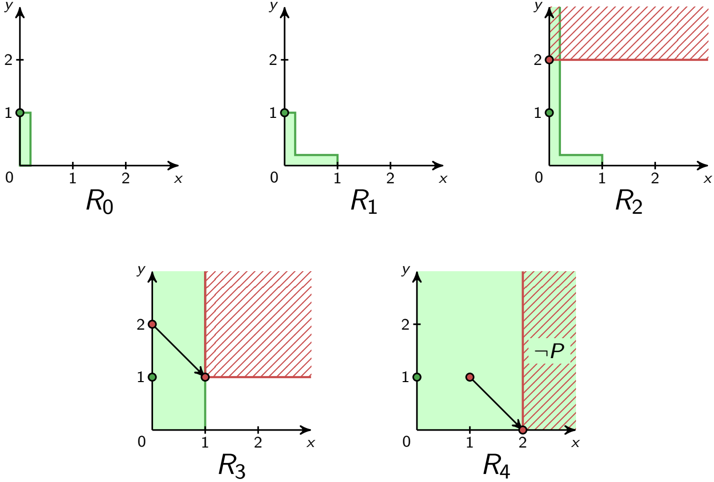

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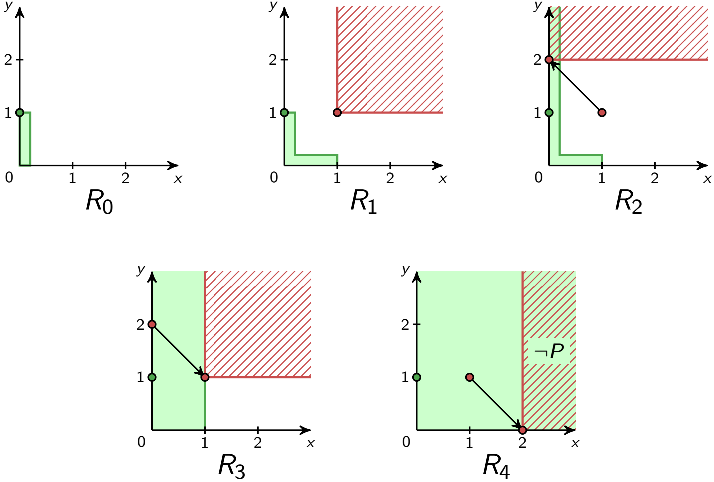

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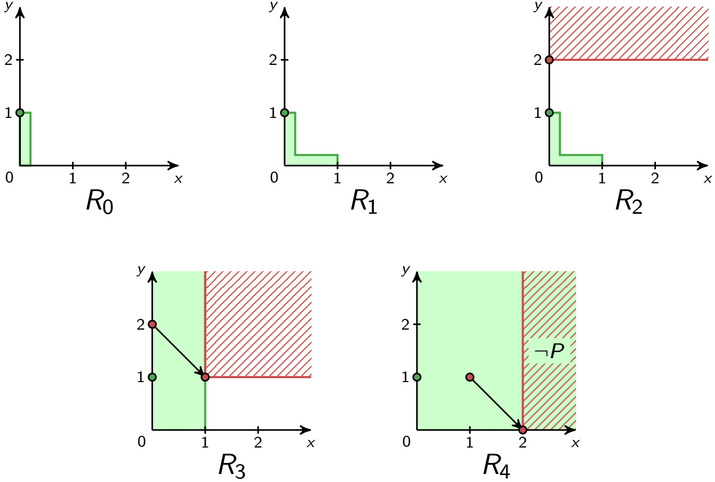

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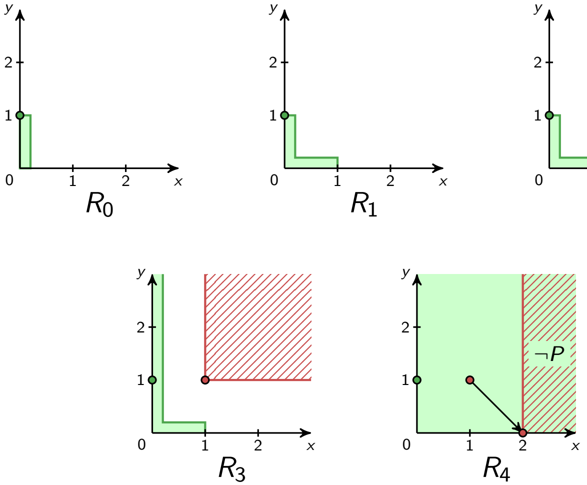

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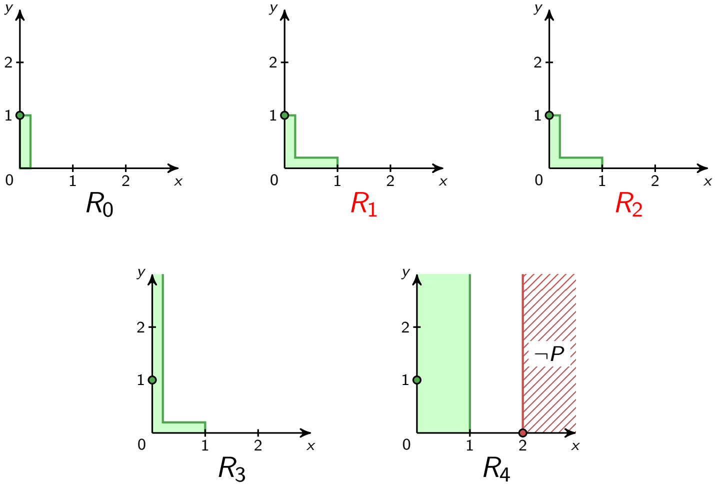

Inductive Invariant

During the execution:

- $I \subseteq R_k$
- $R_k \subseteq R_{k+1}$
- $\text{post}(R_k) \subseteq R_{k+1}$
- $R_k \subseteq P$ for $k < N$

If $R_i = R_{i+1}$:

- $I \subseteq R_i$
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  $\Rightarrow R_i$ is inductive invariant
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Incremental, Inductive Coverability

- Generalizes Aaron Bradley's IC3 to well-structured transition systems (which include Petri nets).
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