To appear in the *Journal of Experimental & Theoretical Artificial Intelligence*
Vol. 00, No. 00, Month 20XX, 1–29

ArgoTriCS - Automated Triangle Construction Solver†

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(*Received 00 Month 20XX; accepted 00 Month 20XX*)

In this paper a method for automatically solving a class of straightedge-and-compass construction problems is proposed. These are the problems where the goal is to construct a triangle given three located points. The method is based on identifying and systematizing geometric knowledge, a specific, restricted search and handling redundant or locus dependent instances. The proposed method is implemented and the current implementation can solve a large number of triangle construction problems. To our knowledge this is the first systematic automated construction solver focused on solving problems from the corpus given. This is also the first approach that consider proving correctness of generated constructions (by using external automated theorem provers).

Keywords: Triangle construction problems, automated deduction in geometry, Wernick's list, search procedure

1. Introduction

The goal of construction problems in geometry is to, given a declarative specification of a figure, determine a corresponding procedural specification based on available construction steps. This procedural specification should be expressed in terms of tools available. In the text that follows the only tools that will be considered are straightedge[1]and compass[2]. This is the oldest and the most studied class of construction problems.

Triangle construction problems are problems in which one has to construct, using straightedge and compass, a triangle that meets given (usually three) constraints [Djorić & Janičić, 2004, Lopes, 1996, Martin, 1998]. The central problem, for a human or for a computer program, in solving construction problems is a huge search space: primitive construction steps can be applied in a number of ways, exploding further along the construction.

Consider, as an illustration, the following problem: *given the points A, O, and H_a, construct a triangle ABC such that O is its circumcenter and H_a is the foot of the altitude from the vertex A.* One possible solution would be to construct the altitude h_a

†Parts of this work at earlier stages were presented at conferences MKM 2012 [Marinković & Janičić, 2012] and ADG 2014 [Marinković et al., 2015].
[1]The notion of "straightedge" is weaker than "ruler", as ruler is assumed to have markings which could be used to make measurements.
[2]By compass, we mean collapsible compass. This means that one cannot use it to "hold" the length while moving one point of the compass to another point. It can be used only to hold the radius while one point of the compass is fixed [Beeson, 2010].

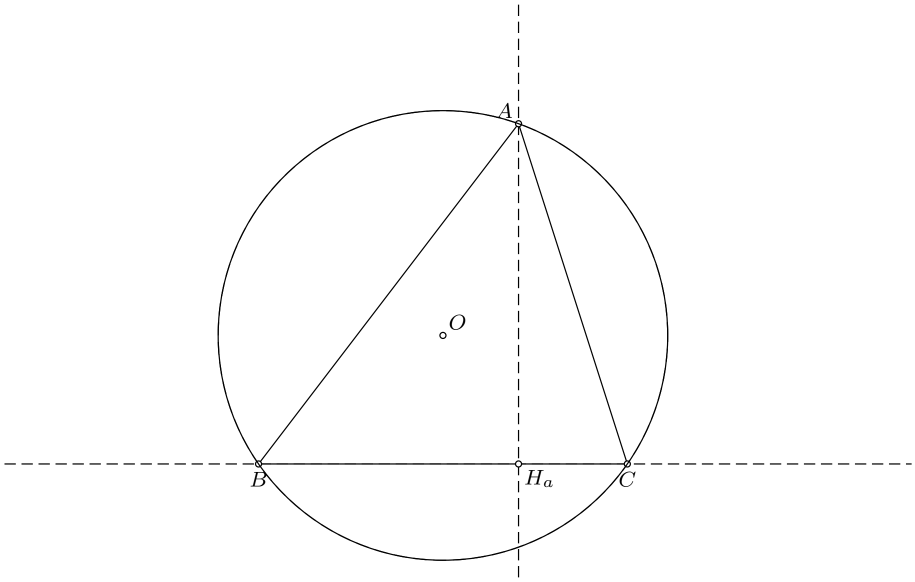


Figure 1.: Construction of a triangle ABC given its vertex A, circumcenter O and the foot of the altitude H_a

through points A and H_a, then construct a line a as a line through point H_a perpendicular to a line h_a, construct a circumcircle of the triangle ABC as a circle with center O through point A, and finally construct vertices B and C as the intersection points of the line a and circumcircle of the triangle ABC (Figure 1). The solution is simple and intuitive. However, if one wants to describe a systematic (e.g., automatic) way for reaching this solution, one should consider a wide range of possibilities. For instance, after constructing the line h_a, one might consider constructing lines AO or OH_a, then instead of constructing perpendicular to the line h_a one might consider constructing a line through point H_a perpendicular to line AO or OH_a, or not even through point H_a but through some other point. Also a circle centered at any of given points, through any other point could be constructed and intersection points of all constructed circles with all constructed lines could be constructed, etc. This simple example illustrates that any systematic procedure focused on solving construction problems can face a huge search space even if only primitive constructions steps that are really needed are considered.

Some of construction problems, for instance, circle squaring, cube duplication, and angle trisection are proved unsolvable [Stewart, 1973] using an algebraic argument. Although the problem of constructibility using straightedge and compass of a figure that can be specified by algebraic equations with coefficients in $\mathbf{Q}$ is decidable [Gao & Chou, 1998b, Guoting, 1992, Lebesgue, 1950], there is no simple and efficient decision procedure and proofs of unsolvability of construction problems are not generated by uniform algorithm, but using *ad hoc* methods.

There are several domains where solving of construction problems is commonly used, like in computer aided design [Hoffmann & Joan-Arinyo, 2002] where a possible task could be to create a system which is capable of correcting an “approximate sketch” entered by a user in order to make it satisfy the set of the constraints given. Also, it can be used in structural chemistry where one wants to know the position of atoms knowing the distances between them [Porta, J.M. et al., 2007]. They can be also useful in robotics, in trilateration problem, where the task is to locate the robot using the distances to three other known points or stations [Thomas & Ros, 2005] and in computer aided surgery [Essert-Villard et al., 2009]. However, solving geometric construction problems plays the most significant role in the area of mathematical education and this role lasts for thousands of years.

There are several approaches for automated solving of construction problems [Gao & Chou, 1998a, Grima & Pace, 2007, Gulwani et al., 2011, Schreck, 1993]. However, most, if not all of them, focus on search procedures and do not focus on finding a small portion of geometry knowledge that is underlying the constructions. We find that it is important to systematize the knowledge relevant for solving construction problems of one corpus of construction problems, since such understanding could help in developing a system that automatically solves this kind of problems.[3]

In this work we focus on two corpora of triangle construction problems and we propose a method for automatically solving geometry construction problems. This method is based on:

- identifying relevant geometry knowledge and its separation into definitions, lemmas and primitive construction;
- guided search which restricts construction of objects only to those objects that could be relevant for a construction;
- appropriate handling redundant and locus depenedent instances of problem.

The implemented system succeeded to solve most of the solvable problems from these two corpora. The same approach can be applied to other sorts of triangle construction problems and, more generally, to other sorts of construction problems. To our knowledge, this is the first approach that consider proving constructions correct.

The implemented system should be useful in mathematical education, in computer-aided learning, for millions of students solving these sorts of problems. It would be also useful in formalization of the mathematical knowledge.

Overview of the paper. In Section 2, construction problem by straightedge and compass is defined and two corpora of triangle construction problems are described. In Section 3, a geometry knowledge needed for solving problems from these two corpora is presented and the proposed solving procedure is described. In Section 4, the corresponding implementation, with few examples, is given. In Section 5, related work is briefly discussed. In Section 6, the status of ongoing work, as well as plans for the future work are described and in Section 7, final conclusions are drawn.

2. Background

In this section we recall some basic notions on geometry construction problems and describe two corpora of construction problems: Wernick's corpus and Connelly's corpus.

2.1. *Constructions by Straightedge and Compass*

A *straightedge-and-compass construction problem* is a problem in which one has to, given a declarative specification of a figure, provide a corresponding – possibly equivalent – procedural specification of the figure based on available construction steps. By a *straightedge-and-compass construction* we will mean a sequence of the following primitive (or elementary) steps:

[3] The existing attempts at manual systematization of triangle construction problems also didn't provide small and clear lists of needed underlying geometry knowledge [Fursenko, 1937a,b, Lopes, 1996].

- construct an arbitrary point (possibly distinct from some given points);
- construct (by *straightedge*) the line passing through two given points;
- construct (by *compass*) the circle centered at some point passing through another point;
- construct an intersection (if it exists) of two circles, two lines, or a line and a circle.

In describing geometrical constructions, both primitive and compound construction steps can be used.

A solution of a geometrical construction problem traditionally includes the following four phases [Adler, 1906, Djorić & Janičić, 2004, Holland, 1992, Lopes, 1996]:

Analysis: In analysis one typically starts from the assumption that a certain geometrical object satisfies the specification Γ and proves that properties Λ enabling the construction also hold;

Construction: In this phase, straightedge-and-compass construction based on the analysis (i.e, on the properties Λ which are proved within it) has to be provided;

Proof: In this phase, it has to be proved that the provided straightedge-and-compass construction meets the given specification, i.e., the conditions Γ;

Discussion: In the discussion, it is considered how many possible solutions to the problem there exist and under which conditions.

2.2. *Corpora of Location Triangle Construction Problems*

In 1982, Wernick presented a list of triangle construction problems [Wernick, 1982]. In each problem, a task is to construct a triangle from three located points selected from the following set of 16 characteristic points:

- A, B, C, O: three vertices and circumcenter;
- M_a, M_b, M_c, G: the side midpoints and centroid;
- H_a, H_b, H_c, H: three feet of altitudes and orthocenter;
- T_a, T_b, T_c, I: three intersection points of the internal angles bisectors with the opposite sides of the triangle, and incenter.

There are 560 triples of the above points, but Wernick's list consists only of 139 significantly different non-trivial problems. The triple (A, B, C) is trivial and, for instance, the problems (A, B, M_a), (A, B, M_b), (B, C, M_b), (B, C, M_c), (A, C, M_a), and (A, C, M_c) are considered to be analogous. Some triples are redundant (e.g., (A, B, M_c) – given points A and B, the point M_c is uniquely determined, so it is redundant in (A, B, M_c)). These triples are annotated **R** in Wernick's list. Some triples are constrained by specific conditions, for instance, in the triple (A, B, O) in order to have a solution, the point O has to belong to the perpendicular bisector of the segment AB, and in that case there are infinitely many solutions. These problems are annotated **L** in Wernick's list. Problems that can be solved by straightedge and compass are annotated **S** and problems that cannot be solved by straightedge and compass are annotated **U**.

Wernick's list left 41 problem unresolved, while the update from 1996 [Meyers, 1996] fixed the status of the problem 102 and left only 20 of them unclassified. In the meanwhile, the problems 90, 109, 110, 111 [Specht, 2009], and 138 [Ustinov, 2009] were proved to be unsolvable. Recently all unresolved problems was proved to be unsolvable [Schreck & Mathis, 2014], except the problem 119 which was proved to be solvable; also, it was proved that the problem 108 was erroneously annotated as unsolvable, instead of solvable [Schreck & Mathis, 2014]. The current status of the problems from Wernick's list is given in Table 1: there are 74 **S** problems, 39 **U** problems, 3 **R** problems, and 23 **L** problems.

1.	A, B, O	L	48.	A, H_a, I	S	95.	M_a, G, T_b	U
2.	A, B, M_a	S	49.	A, H_b, H_c	S	96.	M_a, G, I	S
3.	A, B, M_c	R	50.	A, H_b, H	L	97.	M_a, H_a, H_b	S
4.	A, B, G	S	51.	A, H_b, T_a	S	98.	M_a, H_a, H	L
5.	A, B, H_a	L	52.	A, H_b, T_b	L	99.	M_a, H_a, T_a	L
6.	A, B, H_c	L	53.	A, H_b, T_c	S	100.	M_a, H_a, T_b	U
7.	A, B, H	S	54.	A, H_b, I	S	101.	M_a, H_a, I	S
8.	A, B, T_a	S	55.	A, H, T_a	S	102.	M_a, H_b, H_c	L
9.	A, B, T_c	L	56.	A, H, T_b	U	103.	M_a, H_b, H	S
10.	A, B, I	S	57.	A, H, I	S	104.	M_a, H_b, T_a	S
11.	A, O, M_a	S	58.	A, T_a, T_b	S	105.	M_a, H_b, T_b	S
12.	A, O, M_b	L	59.	A, T_a, I	L	106.	M_a, H_b, T_c	U
13.	A, O, G	S	60.	A, T_b, T_c	S	107.	M_a, H_b, I	U
14.	A, O, H_a	S	61.	A, T_b, I	S	108.	M_a, H, T_a	S
15.	A, O, H_b	S	62.	O, M_a, M_b	S	109.	M_a, H, T_b	U
16.	A, O, H	S	63.	O, M_a, G	S	110.	M_a, H, I	U
17.	A, O, T_a	S	64.	O, M_a, H_a	L	111.	M_a, T_a, T_b	U
18.	A, O, T_b	S	65.	O, M_a, H_b	S	112.	M_a, T_a, I	S
19.	A, O, I	S	66.	O, M_a, H	S	113.	M_a, T_b, T_c	U
20.	A, M_a, M_b	S	67.	O, M_a, T_a	L	114.	M_a, T_b, I	U
21.	A, M_a, G	R	68.	O, M_a, T_b	U	115.	G, H_a, H_b	U
22.	A, M_a, H_a	L	69.	O, M_a, I	S	116.	G, H_a, H	S
23.	A, M_a, H_b	S	70.	O, G, H_a	S	117.	G, H_a, T_a	S
24.	A, M_a, H	S	71.	O, G, H	R	118.	G, H_a, T_b	U
25.	A, M_a, T_a	S	72.	O, G, T_a	U	119.	G, H_a, I	S
26.	A, M_a, T_b	U	73.	O, G, I	U	120.	G, H, T_a	U
27.	A, M_a, I	S	74.	O, H_a, H_b	U	121.	G, H, I	U
28.	A, M_b, M_c	S	75.	O, H_a, H	S	122.	G, T_a, T_b	U
29.	A, M_b, G	S	76.	O, H_a, T_a	S	123.	G, T_a, I	U
30.	A, M_b, H_a	L	77.	O, H_a, T_b	U	124.	H_a, H_b, H_c	S
31.	A, M_b, H_b	L	78.	O, H_a, I	U	125.	H_a, H_b, H	S
32.	A, M_b, H_c	L	79.	O, H, T_a	U	126.	H_a, H_b, T_a	S
33.	A, M_b, H	S	80.	O, H, I	U	127.	H_a, H_b, T_c	U
34.	A, M_b, T_a	S	81.	O, T_a, T_b	U	128.	H_a, H_b, I	U
35.	A, M_b, T_b	L	82.	O, T_a, I	S	129.	H_a, H, T_a	L
36.	A, M_b, T_c	S	83.	M_a, M_b, M_c	S	130.	H_a, H, T_b	U
37.	A, M_b, I	S	84.	M_a, M_b, G	S	131.	H_a, H, I	S
38.	A, G, H_a	L	85.	M_a, M_b, H_a	S	132.	H_a, T_a, T_b	U
39.	A, G, H_b	S	86.	M_a, M_b, H_c	S	133.	H_a, T_a, I	S
40.	A, G, H	S	87.	M_a, M_b, H	S	134.	H_a, T_b, T_c	U
41.	A, G, T_a	S	88.	M_a, M_b, T_a	U	135.	H_a, T_b, I	U
42.	A, G, T_b	U	89.	M_a, M_b, T_c	U	136.	H, T_a, T_b	U
43.	A, G, I	S	90.	M_a, M_b, I	U	137.	H, T_a, I	U
44.	A, H_a, H_b	S	91.	M_a, G, H_a	L	138.	T_a, T_b, T_c	U
45.	A, H_a, H	L	92.	M_a, G, H_b	S	139.	T_a, T_b, I	S
46.	A, H_a, T_a	L	93.	M_a, G, H	S			
47.	A, H_a, T_b	S	94.	M_a, G, T_a	S			

Table 1.: Status of the problems from Wernick's list

Solutions for 59 solvable problems can be found on the Internet [Specht, 2009].

An extended list, involving four additional points:

- E_a, E_b, E_c: three Euler points, which are the midpoints between the vertices and the orthocenter;
- N: the center of the nine-point circle

was presented and partly solved by Connelly [Connelly, 2009]. There are 140 significantly different problems that can be described by adding new points to the set of points from the Wernick's list. In Table 2, the current status of the problems from Connelly's list is given: there are 73 **S** problems, 11 **U** problems, 5 **R** problems, 19 **L** problems, as well as 32 problems with unknown status.

In Wernick's list, the problems (A, B, G) and (A, C, G) are considered analogous and therefore the second one is not even listed. In constrast to this, in our approach the centroid of a triangle is defined as an intersection point of medians $m(AB)$ and $m(AC)$, and the property that the third median $m(BC)$ passes through it is considered a lemma. Therefore, in contrast to Wernick's list, we will consider the problems (A, B, G) and

1.	A, B, E_a	S	48.	E_a, E_b, M_c	S	95.	E_a, I, T_b	
2.	A, B, E_c	S	49.	E_a, E_b, N	L	96.	E_a, M_a, M_b	L
3.	A, B, N	S	50.	E_a, E_b, O	S	97.	E_a, M_a, N	R
4.	A, E_a, E_b	S	51.	E_a, E_b, T_a		98.	E_a, M_a, O	S
5.	A, E_a, G	S	52.	E_a, E_b, T_c	U	99.	E_a, M_a, T_a	S
6.	A, E_a, H	R	53.	E_a, G, H	S	100.	E_a, M_a, T_b	
7.	A, E_a, H_a	L	54.	E_a, G, H_a	S	101.	E_a, M_b, M_c	S
8.	A, E_a, H_b	L	55.	E_a, G, H_b	S	102.	E_a, M_b, N	L
9.	A, E_a, I	S	56.	E_a, G, I		103.	E_a, M_b, O	S
10.	A, E_a, M_a	S	57.	E_a, G, M_a	S	104.	E_a, M_b, T_a	
11.	A, E_a, M_b	S	58.	E_a, G, M_b	S	105.	E_a, M_b, T_b	
12.	A, E_a, N	S	59.	E_a, G, N	S	106.	E_a, M_b, T_c	
13.	A, E_a, O	S	60.	E_a, G, O	S	107.	E_a, N, O	S
14.	A, E_a, T_a	S	61.	E_a, G, T_a		108.	E_a, N, T_a	S
15.	A, E_a, T_b	U	62.	E_a, G, T_b		109.	E_a, N, T_b	
16.	A, E_b, E_c	S	63.	E_a, H, H_a	L	110.	E_a, O, T_a	
17.	A, E_b, G	S	64.	E_a, H, H_b	L	111.	E_a, O, T_b	
18.	A, E_b, H	S	65.	E_a, H, I	S	112.	E_a, T_a, T_b	
19.	A, E_b, H_a	S	66.	E_a, H, M_a	S	113.	E_a, T_b, T_c	
20.	A, E_b, H_b	L	67.	E_a, H, M_b	S	114.	G, H, N	R
21.	A, E_b, H_c	S	68.	E_a, H, N	S	115.	G, H_a, N	S
22.	A, E_b, I		69.	E_a, H, O	S	116.	G, I, N	U
23.	A, E_b, M_a	S	70.	E_a, H, T_a	S	117.	G, M_a, N	S
24.	A, E_b, M_b	S	71.	E_a, H, T_b	U	118.	G, N, O	R
25.	A, E_b, M_c	S	72.	E_a, H_a, H_b	S	119.	G, N, T_a	U
26.	A, E_b, N	S	73.	E_a, H_a, I	S	120.	H, H_a, N	S
27.	A, E_b, O	S	74.	E_a, H_a, M_a	L	121.	H, I, N	U
28.	A, E_b, T_a		75.	E_a, H_a, M_b	S	122.	H, M_a, N	S
29.	A, E_b, T_b		76.	E_a, H_a, N	L	123.	H, N, O	R
30.	A, E_b, T_c		77.	E_a, H_a, O	S	124.	H, N, T_a	U
31.	A, G, N	S	78.	E_a, H_a, T_a	L	125.	H_a, H_b, N	L
32.	A, N, N	S	79.	E_a, H_a, T_b		126.	H_a, I, N	S
33.	A, H_a, N	S	80.	E_a, H_b, H_c	L	127.	H_a, M_a, N	L
34.	A, H_b, N	S	81.	E_a, H_b, I		128.	H_a, M_b, N	L
35.	A, I, N		82.	E_a, H_b, M_a	L	129.	H_a, N, O	S
36.	A, M_a, N	S	83.	E_a, H_b, M_b	S	130.	H_a, N, T_a	S
37.	A, M_b, N	S	84.	E_a, H_b, M_c	S	131.	H_a, N, T_b	
38.	A, N, O	S	85.	E_a, H_b, N	L	132.	I, M_a, N	S
39.	A, N, T_a		86.	E_a, H_b, O	S	133.	I, N, O	U
40.	A, N, T_b		87.	E_a, H_b, T_a		134.	I, N, T_a	
41.	E_a, E_b, E_c	S	88.	E_a, H_b, T_b	U	135.	M_a, M_b, N	L
42.	E_a, E_b, G	S	89.	E_a, H_b, T_c		136.	M_a, N, O	S
43.	E_a, E_b, H	S	90.	E_a, I, M_a	S	137.	M_a, N, T_a	S
44.	E_a, E_b, H_a	S	91.	E_a, I, M_b		138.	M_a, N, T_b	
45.	E_a, E_b, H_c	S	92.	E_a, I, N	S	139.	N, O, T_a	U
46.	E_a, E_b, I	U	93.	E_a, I, O		140.	N, T_a, T_b	
47.	E_a, E_b, M_a	L	94.	E_a, I, T_a				

Table 2.: Status of the problems from Connelly's list

(A, C, G) distinct. For each problem we will determine if it is *symmetric modulo definitions* to some of the previous problems (which is annotated by S_d), or *symmetric modulo definitions and lemmas* (annotated by S_{dl}). For example, the problems (B, M_a, G) and (A, M_b, G) will be considered symmetric modulo definitions, and the problems (A, B, G) and (A, C, G) will be considered symmetric modulo definitions and lemmas.

3. The ArgoTriCS system

ArgoTriCS (**A**utomated **R**easoning **G**r**O**up **Tri**angle **C**onstruction **S**olver) is a tool that, given some background geometrical knowledge, solves automatically a construction problem. Solving of a construction problem includes:

- automated generation of informal description of construction in natural language form;
- automated generation of formal specification of construction using the GCLC language [Janičić, 2006, Janičić & Quaresma, 2006], accompanied by a corresponding

illustration;
- proving construction correct by using automated provers (OpenGeoProver [Marić et al. , 2012] and provers existing within GCLC tool);
- generation of non-degeneracy conditions (NDG conditions) which guarantee that the solution exists.

3.1. *Detecting relevant Definitions, Lemmas, and Construction Primitives*

In order to get solutions more efficiently, after a careful study and manual analysis of available solutions, we came to a relatively small core of geometry knowledge needed for solving almost all problems from these two corpora. The detected knowledge is separated into the set of definitions, lemmas and primitive constructions and a simple algorithm for solving problems from these corpora is developed. This knowledge is minimal in a sense that each of its elements is needed for solving at least one problem, hence there is no proper subset of detected knowledge that would be sufficient for solving the same set of problems.

Consider again the problem from Section 1 (it is the problem 14 from Wernick's list). One solution is as follows: *construct the line h_a through points A and H_a, then construct a line a through point H_a perpendicular to a line h_a, construct a circle $c(O, A)$ with center O through point A and, finally, construct points B and C as the intersection points of the line a and the circle $c(O, A)$.* Notice that this solution implicitly or explicitly uses the following knowledge:

(1) the point O is the circumcenter of the triangle ABC (definition of O);
(2) the point H_a is the foot of the altitude from the point A (definition of H_a);
(3) the circle $c(O, A)$ contains the points B and C (lemma);
(4) the line h_a contains the point A and is perpendicular to the line a (definition of line h_a);
(5) given points X and Y, it is possible to construct a line that contains them both (primitive construction);
(6) given a point X and a line p, it is possible to construct a line q that contains point X and is perpendicular to line p (primitive construction);
(7) given points X and Y it is possible to construct a circle centered at point X that contains point Y (primitive construction);
(8) given a line p and a circle c it is possible to construct their intersection points (primitive construction).

The nature of the above properties is typically not stressed within solutions of construction problems. Following our analysis of the problems from Wernick's and Connelly's corpora, we insist on a clear separation into definitions, lemmas (geometry properties), and construction primitives in the process of solving construction problems.[4] This separation of concepts will be also important for automating the solving process.

We distinguish between two types of definitions:

- *instantiated definitions*: definitions that define one single object;
- *general definitions*: definitions that define one class of objects.

For example, the definition of the orthocenter is instantiated only for the triangle ABC since there is only one relevant orthocenter, while there is a general definition of the line

[4]The complete list of definitions, lemmas and construction primitives used is given in Appendix B.

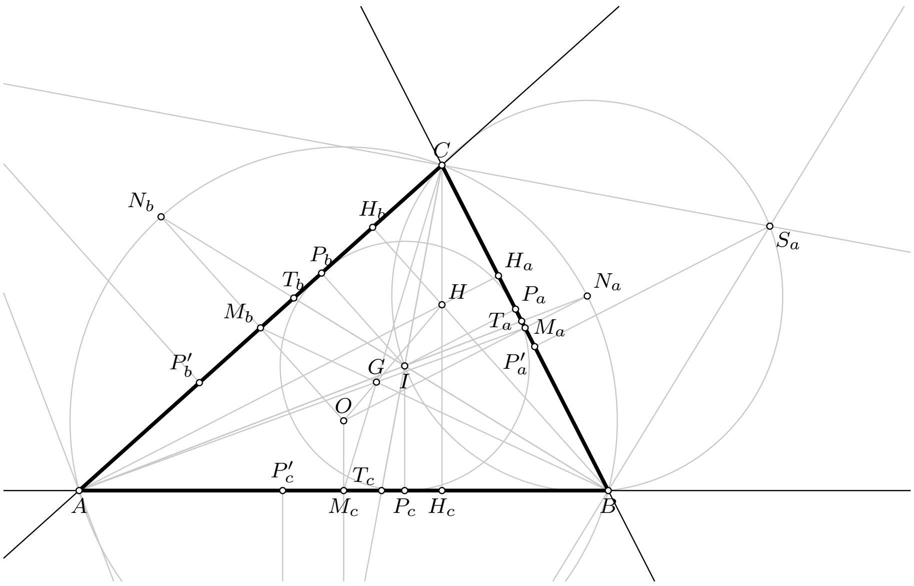


Figure 2.: Points used in solutions to Wernick's problems

that contains two points, since there are many relevant lines through two points. We will try to have most of the definitions instantiated, since in that way the high level of control over the objects introduced in construction is achieved.

In Figure 2 the points used in solving problems from Wernick's list are illustrated. For solving problems from Connelly's corpus seven more points are needed.[5]

Some geometry properties come from definitions, and some do not – these properties will be called lemmas.[6] All identified lemmas are relatively simple and are often taught in primary or secondary schools.

Two types of lemmas are identified:

- *instantiated lemmas*: lemmas that describe properties of one or more fixed objects;
- *general lemmas*: lemmas that are given in an implication form.

Primitive constructions are given in non-instantiated form and they get instantiated during the search for a construction. Each of the construction primitives is accompanied by:

- the set of *non-degeneracy conditions* – conditions which guarantee that the constructed objects indeed exist;
- the set of *determination conditions* – conditions which enable that the constructed objects are uniquely determined.

All of the construction steps identified can be (most of them trivially) expressed in terms of straightedge and compass operations. Still, for practical reasons, we use the set of compound construction steps instead of elementary straightedge and compass operations. These reasons are both more efficient search and simpler, high-level and more intuitive solutions.

[5]These are the three Euler points, the center of the nine-point circle and three more points.

[6]For instance, since the circumcenter O of the triangle ABC is defined as an intersection point of side bisectors $m(AB)$ and $m(AC)$, the property that the bisector of the third side BC passes through the point O is considered a lemma.

The list of the definitions, lemmas and primitive constructions needed was identified through the analysis of particular solutions to some of the construction problems. The analysis was carried out in iterations. The first few problems were solved manually and their solutions were analyzed. The knowledge needed for solving these problems was added to the knowledge base. Then all other problems were tried to be solved by the ArgoTriCS system using this knowledge base. Then again solutions to some of unsolved problems were analyzed and additional knowledge needed was added to the knowledge base, and so on.

3.2. *Overview of the algorithm*

Let a family of construction problems to be solved is given. Solving of problems from this family consists of the following steps:

- generation of all problems from the family according to the set of given objects,
- preprocessing phase, and
- solving phase, which consists of:
  - determining if the problem is symmetric to some of already solved problems,
  - determining if the problem is redundant,
  - determining if the problem is locus dependent,
  - generation of construction,
  - determining when the problem has a solution and how many solutions there are.

3.2.1. Generation of problems

For each of the families of construction problems to be solved a set of given elements is provided and then all distinct triples over that set are formed. The set of given elements for Connelly's corpus is actually an extension of the set of given elements of Wernick's corpus, but, according to Connelly's list, all triples present in Wernick's corpus are excluded from Connelly's list.

3.2.2. Preprocessing phase

Before the solving process starts, the preprocessing phase is performed on the set of general definitions and lemmas. During the preprocessing phase, general definitions are instantiated by all relevant objects and they are added to the knowledge base as derived definitions. Instantiation of general lemmas by all objects satisfying their preconditions is also performed and derived facts are added to the knowledge base as derived lemmas. Preprocessing phase is performed just once.

3.2.3. Solving phase

After the preprocessing phase finishes, the solving phase starts.

3.2.3.1. Determining if the problem is symmetric. Firstly, each problem is tested for symmetry (modulo definitions, or definitions and lemmas) to some other, already solved, problem. At the moment all problems are solved from scratch, but it could be possible to use an information to which problem a considered problem is symmetric and to use its solution in order to generate solutions more efficiently.

3.2.3.2. Determining if the problem is redundant. Then each problem is tested if it is redundant, as described in Section 2.2. A problem is considered *redundant* if one of the given elements can be constructed using other two elements. If the problem is found redundant, the vertex of the triangle which is still not constructed is constructed arbitrarily, and if the construction is not finished yet, search for a construction proceeds in usual manner.

3.2.3.3. Determining if the problem is locus dependent. If the problem is not redundant, then it is tested if one of the given elements belongs to some locus determined by other two elements. If this is true, the problem is declared *locus dependent*, as described in Section 2.2, and construction proceeds in the following manner: that element is chosen arbitrarily such that it belongs to that locus and after that search for a construction advances in usual manner.

3.2.3.4. Generation of construction. If the problem is found neither redundant nor locus dependent, the search for a construction starts. The goal of the search procedure is to reach all points required by the input problem (for instance, for all problems from Wernick's and Connelly's corpora, the goal is the same: construct a triangle ABC, i.e. the points A, B and C). As already said, primitive constructions are given in a generic, non-instantiated form and they get instantiated while seeking for a construction in the following manner: if there is an instantiation such that all objects from the preconditions of the primitive construction are already constructed (or given by a specification of the problem) then the instantiated object from the conclusion is constructed, if not already constructed. The outline of the search procedure is the following: it is iterative and in each step it tries to apply a primitive construction to the known objects (given by the problem specification or already constructed) and if it succeeds, the search restarts from the first primitive construction, in the linear sequential manner. However, the search is restricted to the set of relevant objects, as it will be discussed in the further text. If all required points are constructed, the search stops and the problem is declared *solvable*. If no primitive construction can be applied, the procedure stops with a failure, meaning that the problem is not solvable using given knowledge base.

During the solving process, the sets of all definitions, lemmas and primitive constructions used are maintained, so after the construction succeeds one can precisely determine which knowledge was used for solving that problem.

The efficiency of solving, and also the found solution may depend on the order in which the primitive constructions are listed. The efficiency of this basic procedure is improved by using some additional techniques:

Restriction on the objects being constructed

Objects that are being constructed are limited to the set of objects that are relevant for some object which is still not constructed, i.e. objects appearing in a definition of the object which is not already constructed, or a lemma which involves an object not already constructed. For example, let us consider the primitive construction stating that for two given points it is possible to construct the midpoint of the segment they constitute. If there would be no restrictions, the midpoint would be constructed for each segment with endpoints among any of two constructed points, while many of them would not be used anywhere further; furthermore constructed midpoints would be considered as an ending points of the segments and their midpoints would be constructed. To avoid this undesired effect, this rule would be applied only to a segment for which holds that its midpoint occurs in some of the definitions or lemmas that also involve an object that is not already constructed.

For instance, if the point M_c occured only in the definition of the side bisector of the segment AB, within the property $inc(M_c, side\_bis(A, B))$ (where $inc()$ denotes an incidence predicate and $side\_bis()$ a side bisector of the given segment), and the side bisector of the segment AB had been already constructed, then there would be no use in constructing the point M_c. However, if the point M_c occured also in the lemma: $\overrightarrow{AM_c}/\overrightarrow{AB} = 1/2$ and one of the points A and B had not been already constructed, then it could be useful to construct the point M_c.

This limitation can reduce the search time significantly, as well as the length of generated construction.

Early prunning of inapplicable primitive constructions

While searching for an applicable primitive construction and testing for applicability the primitive constructions that are preeceding the one that have been last used, the following restriction is performed: at least one of the objects from its list of the known objects has to be instantiated by the object that has been constructed in the previous construction step. This is correct since these construction primitives were not applicable before the last construction step, therefore they could become applicable only for the newly constructed objects.

For instance, if the last used primitive construction was the construction C_i which constructs intersection points of the two known circles, and if the primitive construction C_j which constructs an intersection point of two known lines preceeds it, then after application of the construction C_i there is no need to test the construction C_j for applicability, since it was not applicable before the last construction step, and no new lines were constructed in the meanwhile.

Ordering on primitive constructions

For determining an ordering in which primitive constructions should be given, the following heuristic is used: the primitive constructions that are used more frequently are preceeding those that are used for solving just a couple of instances of construction problems. The ones that are more frequently used are identified through preliminary experimenting with different orderings and counting how many times each one was used.

Dealing with different ways of specifying the same object

When testing if the instantiated object from the conclusion of the primitive construction is not already constructed, it is tested for different ways of its representation.

For example, if the premises of the primitive construction can be satisifed and it should construct the line that is an image of the line AB in homothety with center at the point C with a coefficient $1/2$, and the line M_aM_b is already constructed, the procedure will conclude that this primitive construction is not applicable.

3.2.3.5. Discussion phase. For problems that are found solvable it is important to determine when the constructed objects exist and what is the number of solutions. As already mentioned, along with the construction sequence, the set of non-degeneracy conditions is maintained and it is used to state when the constructed objects indeed exist [Marinković et al., 2015]. Considering the number of solutions, it is calculated as the product of number of possible choices for each of the objects constructed during the construction. For instance, if a point is constructed as the intersection point of two circles that intersect, then there are two different ways how to construct it. In case when a point is constructed as an arbitrary point on the line/circle, there is an infinite number of choices.

4. Implementation

The system ArgoTriCS is implemented in PROLOG[7] and, together with the part of code that specifies knowledge base incorporated in the system, it has around 3000 lines of code.

Even though the search is directed and limited to the set of relevant objects, during the search for a construction some irrelevant construction steps are generated. When the construction is generated, all irrelevant steps are eliminated from the final "clean" construction. So once the construction is found and simplified, it can be exported to different formats. Currently export to natural language form (in English language, in LaTeX format), as well as export to the GCLC language are supported. For example, a generated construction for problem 32: (A, O, H_a) in natural language follows:

Problem 32: Given a point A, a point O, and a point H_a construct the triangle ABC.
Construction:

(1) Using the point A and the point H_a construct a line h_a (rule W02);
 % DET: points A and H_a are not the same;
(2) Using the point A and the point O construct a circle $k(O, C)$ (rule W06);
 % NDG: points A and O are not the same;
(3) Using the point H_a and the line h_a construct a line a (rule W10);
(4) Using the circle $k(O, C)$ and the line a construct a point C and a point B (rule W04);
 % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same.
Determination conditions: points A and H_a are not the same.
Rules used: [W02,W04,W06,W10a]
Lemmas and definitions used: [D5,D8,D26,GD01,L11,L12]

Formal specification of generated construction, along with its illustration, is generated automatically using geometrical language GCLC [Janičić, 2006, Janičić & Quaresma, 2006, Janičić, 2010]. Beside construction, a specification of input to the provers integrated in GCLC tool can be generated.

Automatically generated output in GCLC language for the problem 32 follows:

```
% free points
point A 80 95
point O 65 51.14
point H_a 80 40
color 220 0 0
fontsize 9
cmark_r A
cmark_r O
cmark_r H_a
color 0 0 0
fontsize 8
```

[7] The source code is available at http://argo.matf.bg.ac.rs/?content=downloads.

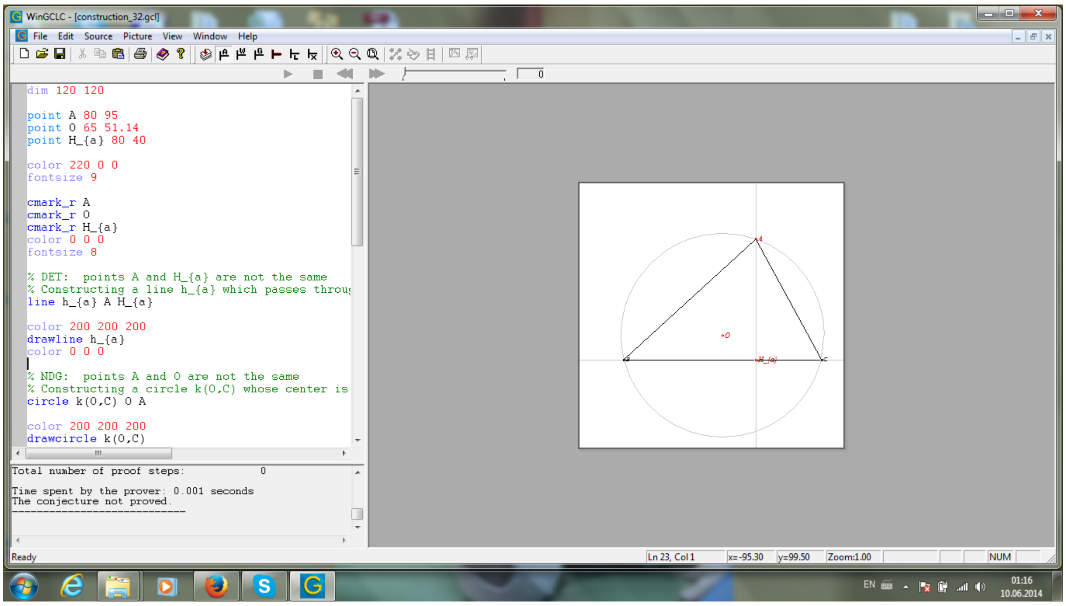


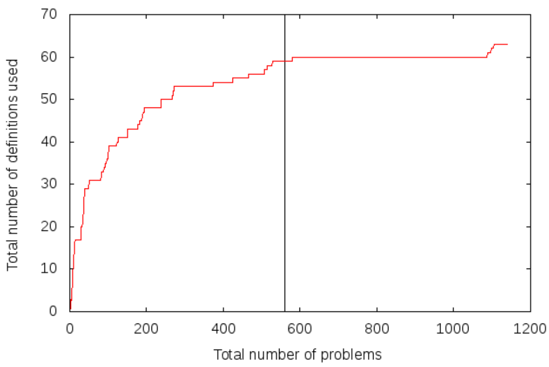
Figure 3.: Illustration of generated construction for the problem A, O, H_a

```
% synthesized construction
% DET: points A and H_a are not the same
% Constructing a line h_a which passes through point A and point H_a
line h_a A H_a
color 200 200 200
drawline h_a
color 0 0 0
% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes
through point A
circle k(O,C) O A
color 200 200 200
drawcircle k(O,C)
color 0 0 0
% Constructing a line a which is perpendicular to line h_a and which passes
through point H_a
perp a H_a h_a
color 200 200 200
drawline a
color 0 0 0
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
drawsegment A B
drawsegment A C
drawsegment B C
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and
O are not the same
% Determination conditions: points A and H_a are not the same
```

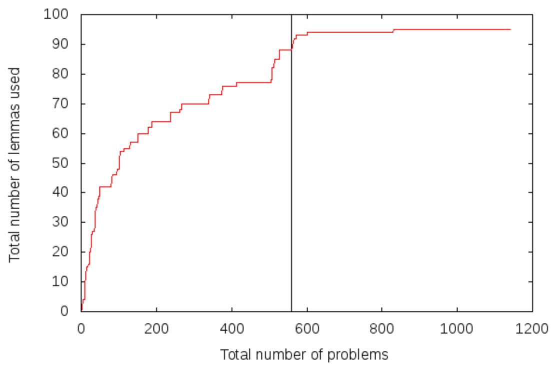
The illustration of the generated construction using GCLC tool is given in Figure 3.

The longest "clean" construction generated consists of 19 primitive construction steps (for problem 1271: E_a, I, M_a from Connelly's corpus). Most of generated constructions are the same as the ones that can be found in the literature. However, for problems with several different solutions, the one found by the system depends on the order of available primitive constructions/definitions/lemmas.

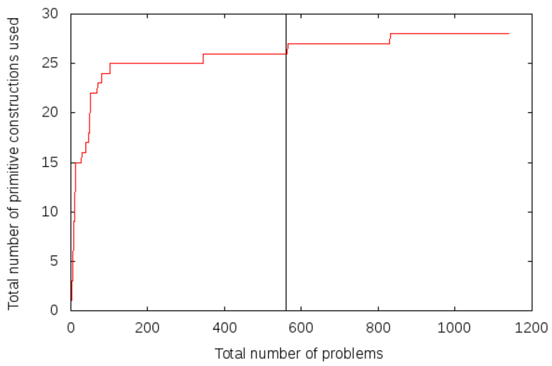
Solving times differ a lot: they span from couple of miliseconds to more than one hour. The hardest instances are the ones that are locus dependent, since then one has to discover a locus that one of the points given belongs to.



(a)



(b)



(c)

Figure 4.: Number of (a) definitions, (b) lemmas, and (c) primitive constructions used in problems from Wernick's and Connelly's corpora

The program can solve 66 out of 74 solvable problems from Wernick's list and 62 out of 73 solvable problems from Connelly's list. It can detect all redundant and locus dependent problems, as well as problems symmetric to another ones. According to stricter consideration of the corpus (discussed at the end of Section 2.2), ArgoTriCS was used for solving all 560 triples of points from Wernick's list and it identified 268 of them solvable, 93 of them locus dependent, 7 of them redundant while it did not solve 192 triples. 166 of them are proved (using algebraic provers) unsolvable, while some of them require some more knowledge to be added to the system. For the 580 triples of points from Connelly's list, the performance of ArgoTriCS is the following: 223 triples were annotated solvable, 84 locus dependent, 9 of them redundant, while 264 of them the system could not solve using background knowledge given. Recall that there are many instances in Connelly's list that are proved unsolvable or whose status is still not determined. Some of the problem were identifed symmetric modulo definitions or modulo definitions and lemmas. Tables with detailed statistics are given in Appendix A. Note that for some problems that are proved solvable, the system ArgoTriCS was not able to find a solution, therefore statuses of these problems are marked **U** in tables in the appendix, although their actual statuses are marked **S** in Tables 1 and 2.

We did not succeed to solve some of the problems that are proved solvable (the list of statuses of the problems given online does not offer solutions to some of the problems that are annotated solvable [Specht, 2009]). Also, we could not find their solutions in the literature (here we don't count solutions derivable from the algebraic proofs of solvability). Some of the people working in this field failed to solve them, too. We plan to continue working on these instances.

An analysis of the number of definitions/lemmas/construction steps needed for solving problems from these two corpora was carried out. In Figure 4a the number of definitions used for solving first N problems from Wernick's and Connelly's corpora is given ($0 < N \leqslant 560$ corresponds to problems from Wernick's corpus, while $561 \leqslant N \leqslant 1140$ corresponds to problems from Connelly's corpus and corpora are visually separated by a horizontal line). The total number of definitions used grows fast at start, but then this growth becomes smaller and smaller, and at the end it is close to a constant function, i.e. the number of new definitions needed becomes very small. The similar trend holds for the total number of lemmas and primitive constructions used, and corresponding statistics are given in Figure 4b and Figure 4c. It is especially important to point out that problems from Connelly's corpus needs quite small portion of new knowledge to be added to the knowledge needed for Wernick's corporus.

5. Related Work

Despite a long tradition of solving geometric construction problems using straightedge and compass, their automated solving is barely mentioned in the literature. There are quite few papers about automation of solving construction problems in geometry using geometric approach [Gulwani et al., 2011, Marinković & Janičić, 2012, Schreck, 1994] and few papers on algebraic solving of the same problem [Gao & Chou, 1998b, Guoting, 1992].

In 1994. Schreck developed a system Progé for solving construction problems in PROLOG ([Schreck, 1994]). In contrast to our approach where the goal is to consider well defined corpus with finite number of construction problems, Progé represents general framework in which expert can project geometric universe where the considered problem belongs. This system does not involve neither definitions nor lemmas, but uses types for typing objects, functional and predicative symbols, as well as an axiom set. In case when functional symbols should be interpreted as multifunctions, as a result a list is obtained. Progé deals with constructibility only, hence the questions of unconstructibility can not be solved using it.

The approach developed by Gao and Chou [Gao & Chou, 1998b] assigns coordinates to all points and then transforms the set of geometric constraints into a set of equations. Afterwards Wu's method [Wu, 1978] or Gröbner basis method [Buchberger, 1965] is used for triangulation of obtained system since these types of systems are much easier to solve. In this way the locations of unknown objects are determined on the basis of location of known objects, set of given constraints and some implicit information derived from given set of constraints.

Most of existing approaches for automated solving of geometric construction problems does not even consider proving of correctness of generated solution. For example, a method developed by Gulwani [Gulwani et al., 2011] is derived from the general method for testing and synthesis of pieces of software. Method comes to a construction using probabilistic approach for finding a particular solution which is then used for guiding a search through a huge space of formal function terms. For this kind of method a proof of correctness is really required.

ArgoTriCS solved all triangle construction problems from three located points (4 out of 25) considered by Gulwani et. al; it can currently solve only a small fraction of 135 problems considered by Gao and Chou, since most of them are not triangle construction problems, and even when they are triangle problems, they are not kind of problems where locations of three points are given (constraints often involve distances and/or angles).

6. Ongoing and Future Work

The presented system synthesizes a construction. An external automated theorem prover can be used to prove that the construction meets the specification. Generally, solutions to construction problems are not correct by construction, thus the proof of correctness of generated construction is really needed. Generated constructions are being proved correct using provers available within the GCLC tool (the tool provide support for three methods for automated theorem proving in geometry: the area method, Wu's method, and the Gröbner bases method) [Janičić, 2006, Janičić & Quaresma, 2006] and OpenGeoProver [Marić et al. , 2012]. These provers as an output provide proof objects, as well as non-degeneracy conditions. The conjecture that is being proved is that the points that are given by the problem setting are indeed the corresponding points of the constructed triangle ABC (for example, that the point O is indeed the circumcenter of the triangle ABC). From this follows that the constructed triangle satisifes the specification of the given construction problem.

For instance, the generated output in GCLC language from the Section 4 for the problem 32, in order to prove generated construction correct, would be supplemented by the following piece of code:

```
% verification
line _b A C
towards _M_b C A 0.5
line _a B C
towards _M_a B C 0.5
perp _m_a _M_a _a
perp _m_b _M_b _b
intersec _O _m_a _m_b
perp _h_a A _a
intersec _H_a _a _h_a
prove identical O _O
prove identical H_a _H_a
```

OpenGeoProver proved the corresponding conjecture for the problem 32 in 1.2s. As a non-degeneracy conditions it returned: points A and H_a are not identical, points B and C are not identical, points A, B and C are not collinear and the line through points A and O is not perpendicular to the line through points B and C.

It is important to notice that none of the previous systems considered proving constructions correct.

We are planning to use proof assistants and our automated theorem prover for coherent logic ArgoCLP [Stojanović et al., 2011] for proving that the constructed objects indeed exist under generated non-degenerate conditions. This would lead to a completely machine verifiable solutions of triangle construction problems.

Apart from detecting high-level lemmas and rules needed, we will try to more deeply explore these lemmas and rules. All lemmas used can be proved using a Hilbert's style geometry axioms or automatically – using algebraic theorem provers. One of the plans for the future work is to derive all lemmas used in the system from (suitable) axioms and from elementary straightedge and compass construction steps.

We plan to work on other corpora of triangle construction problems as well and detect new portion of knowledge needed for their solving. We believe that it would be quite small.

7. Conclusions

In this paper we described a practical system for solving most of the problems from Wernick's and Connelly's corpora. To our knowledge, this is the first systematic approach to deal with corpora of construction problems. It uses systematized underlying geometric knowledge, consisting of relatively small set of definitions, lemmas, and primitive constructions. Generated constructions can be verified using external automated theorem provers. We expect that small additions to the geometry knowledge presented here would enable solving many (or most) of triangle construction problems appearing in the literature.

Funding

This work was partially supported by the Serbian Ministry of Education, Science and Technological Development under Grant 174021.

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Appendix A. Lists of the statuses of the problems from Wernick's and Connelly's corpus determined by the system ArgoTriCS

Table A1.: Status of the problems from Wernick's corpus: S denotes that the problem is solvable, L that problem is locus dependent, R that it is redundant, U that it could not be solved by the system ArgoTriCS, $S_d(N)$ denotes that the problem is symmetric modulo definitions to the problem numbered by N, and $S_{dl}(N)$ that it is symmetric modulo definitions and lemmas to the problem numbered by N

1.	A, B, C	S	2.	A, B, O	L	3.	A, B, M_a	S
4.	A, B, M_b	S ($S_d(3)$)	5.	A, B, M_c	R	6.	A, B, G	S
7.	A, B, H_a	L	8.	A, B, H_b	L ($S_d(7)$)	9.	A, B, H_c	L
10.	A, B, H	S	11.	A, B, T_a	S	12.	A, B, T_b	S ($S_d(11)$)
13.	A, B, T_c	L	14.	A, B, I	S	15.	A, C, O	L ($S_{dl}(2)$)
16.	A, C, M_a	S ($S_d(3)$)	17.	A, C, M_b	R ($S_d(5)$)	18.	A, C, M_c	S ($S_d(3)$)
19.	A, C, G	S ($S_{dl}(6)$)	20.	A, C, H_a	L ($S_d(7)$)	21.	A, C, H_b	L ($S_d(9)$)
22.	A, C, H_c	L ($S_d(7)$)	23.	A, C, H	S ($S_{dl}(10)$)	24.	A, C, T_a	S ($S_d(11)$)
25.	A, C, T_b	L ($S_d(13)$)	26.	A, C, T_c	S ($S_d(11)$)	27.	A, C, I	S ($S_{dl}(14)$)
28.	A, O, M_a	S	29.	A, O, M_b	L	30.	A, O, M_c	L ($S_{dl}(29)$)
31.	A, O, G	S	32.	A, O, H_a	S	33.	A, O, H_b	S
34.	A, O, H_c	S ($S_{dl}(33)$)	35.	A, O, H	S	36.	A, O, T_a	S
37.	A, O, T_b	S	38.	A, O, T_c	S ($S_{dl}(37)$)	39.	A, O, I	S
40.	A, M_a, M_b	S	41.	A, M_a, M_c	S ($S_d(40)$)	42.	A, M_a, G	R
43.	A, M_a, H_a	L	44.	A, M_a, H_b	S	45.	A, M_a, H_c	S ($S_d(44)$)
46.	A, M_a, H	S	47.	A, M_a, T_a	S	48.	A, M_a, T_b	U
49.	A, M_a, T_c	U ($S_d(48)$)	50.	A, M_a, I	S	51.	A, M_b, M_c	S
52.	A, M_b, G	S	53.	A, M_b, H_a	L	54.	A, M_b, H_b	L
55.	A, M_b, H_c	L	56.	A, M_b, H	S	57.	A, M_b, T_a	S
58.	A, M_b, T_b	L	59.	A, M_b, T_c	S	60.	A, M_b, I	S
61.	A, M_c, G	S ($S_{dl}(52)$)	62.	A, M_c, H_a	L ($S_d(53)$)	63.	A, M_c, H_b	L ($S_d(55)$)
64.	A, M_c, H_c	L ($S_d(54)$)	65.	A, M_c, H	S ($S_{dl}(56)$)	66.	A, M_c, T_a	S ($S_d(57)$)
67.	A, M_c, T_b	S ($S_d(59)$)	68.	A, M_c, T_c	L ($S_d(58)$)	69.	A, M_c, I	S ($S_{dl}(60)$)
70.	A, G, H_a	L	71.	A, G, H_b	S	72.	A, G, H_c	S ($S_{dl}(71)$)
73.	A, G, H	S	74.	A, G, T_a	S	75.	A, G, T_b	U
76.	A, G, T_c	U ($S_{dl}(75)$)	77.	A, G, I	S	78.	A, H_a, H_b	S
79.	A, H_a, H_c	S ($S_d(78)$)	80.	A, H_a, H	L	81.	A, H_a, T_a	L
82.	A, H_a, T_b	S	83.	A, H_a, T_c	S ($S_d(82)$)	84.	A, H_a, I	S
85.	A, H_b, H_c	S	86.	A, H_b, H	L	87.	A, H_b, T_a	S
88.	A, H_b, T_b	L	89.	A, H_b, T_c	S	90.	A, H_b, I	S
91.	A, H_c, H	L ($S_{dl}(86)$)	92.	A, H_c, T_a	S ($S_d(87)$)	93.	A, H_c, T_b	S ($S_d(89)$)
94.	A, H_c, T_c	L ($S_d(88)$)	95.	A, H_c, I	S ($S_{dl}(90)$)	96.	A, H, T_a	S
97.	A, H, T_b	U	98.	A, H, T_c	U ($S_{dl}(97)$)	99.	A, H, I	U
100.	A, T_a, T_b	S	101.	A, T_a, T_c	S ($S_d(100)$)	102.	A, T_a, I	L
103.	A, T_b, T_c	S	104.	A, T_b, I	S	105.	A, T_c, I	S ($S_{dl}(104)$)
106.	B, C, O	L ($S_{dl}(2)$)	107.	B, C, M_a	R ($S_d(5)$)	108.	B, C, M_b	S ($S_d(3)$)
109.	B, C, M_c	S ($S_d(3)$)	110.	B, C, G	S ($S_{dl}(6)$)	111.	B, C, H_a	L ($S_d(9)$)
112.	B, C, H_b	L ($S_d(7)$)	113.	B, C, H_c	L ($S_d(7)$)	114.	B, C, H	S ($S_{dl}(10)$)
115.	B, C, T_a	L ($S_d(13)$)	116.	B, C, T_b	S ($S_d(11)$)	117.	B, C, T_c	S ($S_d(11)$)
118.	B, C, I	S ($S_{dl}(14)$)	119.	B, O, M_a	L ($S_d(29)$)	120.	B, O, M_b	S ($S_d(28)$)
121.	B, O, M_c	L ($S_{dl}(29)$)	122.	B, O, G	S ($S_d(31)$)	123.	B, O, H_a	S ($S_d(33)$)
124.	B, O, H_b	S ($S_d(32)$)	125.	B, O, H_c	S ($S_{dl}(33)$)	126.	B, O, H	S ($S_d(35)$)
127.	B, O, T_a	S ($S_d(37)$)	128.	B, O, T_b	S ($S_d(36)$)	129.	B, O, T_c	S ($S_{dl}(37)$)
130.	B, O, I	S ($S_d(39)$)	131.	B, M_a, M_b	S ($S_d(40)$)	132.	B, M_a, M_c	S ($S_d(51)$)
133.	B, M_a, G	S ($S_d(52)$)	134.	B, M_a, H_a	L ($S_d(54)$)	135.	B, M_a, H_b	L ($S_d(53)$)
136.	B, M_a, H_c	L ($S_d(55)$)	137.	B, M_a, H	S ($S_d(56)$)	138.	B, M_a, T_a	L ($S_d(58)$)
139.	B, M_a, T_b	S ($S_d(57)$)	140.	B, M_a, T_c	S ($S_d(59)$)	141.	B, M_a, I	S ($S_d(60)$)
142.	B, M_b, M_c	S ($S_d(40)$)	143.	B, M_b, G	R ($S_d(42)$)	144.	B, M_b, H_a	S ($S_d(44)$)

Table A1: *Status of the problems from Wernick's corpus - continued*

145.	B, M_b, H_b	L (S_d(43))	146.	B, M_b, H_c	S (S_d(44))	147.	B, M_b, H	S (S_d(46))
148.	B, M_b, T_a	U (S_d(48))	149.	B, M_b, T_b	S (S_d(47))	150.	B, M_b, T_c	U (S_d(48))
151.	B, M_b, I	S (S_d(50))	152.	B, M_c, G	S (S_{dl}(52))	153.	B, M_c, H_a	L (S_d(55))
154.	B, M_c, H_b	L (S_d(53))	155.	B, M_c, H_c	L (S_d(54))	156.	B, M_c, H	S (S_{dl}(56))
157.	B, M_c, T_a	S (S_d(59))	158.	B, M_c, T_b	S (S_d(57))	159.	B, M_c, T_c	L (S_d(58))
160.	B, M_c, I	S (S_{dl}(60))	161.	B, G, H_a	S (S_d(71))	162.	B, G, H_b	L (S_d(70))
163.	B, G, H_c	S (S_{dl}(71))	164.	B, G, H	S (S_d(73))	165.	B, G, T_a	U (S_d(75))
166.	B, G, T_b	S (S_d(74))	167.	B, G, T_c	U (S_{dl}(75))	168.	B, G, I	S (S_d(77))
169.	B, H_a, H_b	S (S_d(78))	170.	B, H_a, H_c	S (S_d(85))	171.	B, H_a, H	L (S_d(86))
172.	B, H_a, T_a	L (S_d(88))	173.	B, H_a, T_b	S (S_d(87))	174.	B, H_a, T_c	S (S_d(89))
175.	B, H_a, I	S (S_d(90))	176.	B, H_b, H_c	S (S_d(78))	177.	B, H_b, H	L (S_d(80))
178.	B, H_b, T_a	S (S_d(82))	179.	B, H_b, T_b	L (S_d(81))	180.	B, H_b, T_c	S (S_d(82))
181.	B, H_b, I	S (S_d(84))	182.	B, H_c, H	L (S_{dl}(86))	183.	B, H_c, T_a	S (S_d(89))
184.	B, H_c, T_b	S (S_d(87))	185.	B, H_c, T_c	L (S_d(88))	186.	B, H_c, I	S (S_{dl}(90))
187.	B, H, T_a	U (S_d(97))	188.	B, H, T_b	S (S_d(96))	189.	B, H, T_c	U (S_{dl}(97))
190.	B, H, I	U (S_d(99))	191.	B, T_a, T_b	S (S_d(100))	192.	B, T_a, T_c	U (S_d(103))
193.	B, T_a, I	S (S_d(104))	194.	B, T_b, T_c	S (S_d(100))	195.	B, T_b, I	L (S_d(102))
196.	B, T_c, I	S (S_{dl}(104))	197.	C, O, M_a	L (S_{dl}(29))	198.	C, O, M_b	L (S_{dl}(29))
199.	C, O, M_c	S (S_{dl}(28))	200.	C, O, G	S (S_{dl}(31))	201.	C, O, H_a	S (S_{dl}(33))
202.	C, O, H_b	S (S_{dl}(33))	203.	C, O, H_c	S (S_{dl}(32))	204.	C, O, H	S (S_{dl}(35))
205.	C, O, T_a	S (S_{dl}(37))	206.	C, O, T_b	S (S_{dl}(37))	207.	C, O, T_c	S (S_{dl}(36))
208.	C, O, I	S (S_{dl}(39))	209.	C, M_a, M_b	S (S_d(51))	210.	C, M_a, M_c	S (S_d(40))
211.	C, M_a, G	S (S_{dl}(52))	212.	C, M_a, H_a	L (S_d(54))	213.	C, M_a, H_b	L (S_d(55))
214.	C, M_a, H_c	L (S_d(53))	215.	C, M_a, H	S (S_{dl}(56))	216.	C, M_a, T_a	L (S_d(58))
217.	C, M_a, T_b	S (S_d(59))	218.	C, M_a, T_c	S (S_d(57))	219.	C, M_a, I	S (S_{dl}(60))
220.	C, M_b, M_c	S (S_d(40))	221.	C, M_b, G	S (S_{dl}(52))	222.	C, M_b, H_a	L (S_d(55))
223.	C, M_b, H_b	L (S_d(54))	224.	C, M_b, H_c	L (S_d(53))	225.	C, M_b, H	S (S_{dl}(56))
226.	C, M_b, T_a	S (S_d(59))	227.	C, M_b, T_b	L (S_d(58))	228.	C, M_b, T_c	S (S_d(57))
229.	C, M_b, I	S (S_{dl}(60))	230.	C, M_c, G	R (S_{dl}(42))	231.	C, M_c, H_a	S (S_d(44))
232.	C, M_c, H_b	S (S_d(44))	233.	C, M_c, H_c	L (S_d(43))	234.	C, M_c, H	S (S_{dl}(46))
235.	C, M_c, T_a	U (S_d(48))	236.	C, M_c, T_b	U (S_d(48))	237.	C, M_c, T_c	S (S_d(47))
238.	C, M_c, I	S (S_{dl}(50))	239.	C, G, H_a	S (S_{dl}(71))	240.	C, G, H_b	S (S_{dl}(71))
241.	C, G, H_c	L (S_{dl}(70))	242.	C, G, H	S (S_{dl}(73))	243.	C, G, T_a	U (S_{dl}(75))
244.	C, G, T_b	U (S_{dl}(75))	245.	C, G, T_c	S (S_{dl}(74))	246.	C, G, I	S (S_{dl}(77))
247.	C, H_a, H_b	S (S_d(85))	248.	C, H_a, H_c	S (S_d(78))	249.	C, H_a, H	L (S_{dl}(86))
250.	C, H_a, T_a	L (S_d(88))	251.	C, H_a, T_b	S (S_d(89))	252.	C, H_a, T_c	S (S_d(87))
253.	C, H_a, I	S (S_{dl}(90))	254.	C, H_b, H_c	S (S_d(78))	255.	C, H_b, H	L (S_{dl}(86))
256.	C, H_b, T_a	S (S_d(89))	257.	C, H_b, T_b	L (S_d(88))	258.	C, H_b, T_c	S (S_d(87))
259.	C, H_b, I	S (S_{dl}(90))	260.	C, H_c, H	L (S_{dl}(80))	261.	C, H_c, T_a	S (S_d(82))
262.	C, H_c, T_b	S (S_d(82))	263.	C, H_c, T_c	L (S_d(81))	264.	C, H_c, I	S (S_{dl}(84))
265.	C, H, T_a	U (S_{dl}(97))	266.	C, H, T_b	U (S_{dl}(97))	267.	C, H, T_c	S (S_{dl}(96))
268.	C, H, I	U (S_{dl}(99))	269.	C, T_a, T_b	U (S_d(103))	270.	C, T_a, T_c	S (S_d(100))
271.	C, T_a, I	S (S_{dl}(104))	272.	C, T_b, T_c	S (S_d(100))	273.	C, T_b, I	S (S_{dl}(104))
274.	C, T_c, I	L (S_{dl}(102))	275.	O, M_a, M_b	S	276.	O, M_a, M_c	S (S_{dl}(275))
277.	O, M_a, G	S	278.	O, M_a, H_a	L	279.	O, M_a, H_b	S
280.	O, M_a, H_c	S (S_{dl}(279))	281.	O, M_a, H	S	282.	O, M_a, T_a	L
283.	O, M_a, T_b	U	284.	O, M_a, T_c	U (S_{dl}(283))	285.	O, M_a, I	U
286.	O, M_b, M_c	S (S_{dl}(275))	287.	O, M_b, G	S (S_d(277))	288.	O, M_b, H_a	S (S_d(279))
289.	O, M_b, H_b	L (S_d(278))	290.	O, M_b, H_c	S (S_{dl}(279))	291.	O, M_b, H	S (S_d(281))
292.	O, M_b, T_a	U (S_d(283))	293.	O, M_b, T_b	L (S_d(282))	294.	O, M_b, T_c	U (S_{dl}(283))
295.	O, M_b, I	U (S_d(285))	296.	O, M_c, G	S (S_{dl}(277))	297.	O, M_c, H_a	S (S_{dl}(279))
298.	O, M_c, H_b	S (S_{dl}(279))	299.	O, M_c, H_c	L (S_{dl}(278))	300.	O, M_c, H	S (S_{dl}(281))
301.	O, M_c, T_a	U (S_{dl}(283))	302.	O, M_c, T_b	U (S_{dl}(283))	303.	O, M_c, T_c	L (S_{dl}(282))
304.	O, M_c, I	U (S_{dl}(285))	305.	O, G, H_a	S	306.	O, G, H_b	S (S_d(305))
307.	O, G, H_c	S (S_{dl}(305))	308.	O, G, H	R	309.	O, G, T_a	U
310.	O, G, T_b	U (S_d(309))	311.	O, G, T_c	U (S_{dl}(309))	312.	O, G, I	U
313.	O, H_a, H_b	U	314.	O, H_a, H_c	U (S_{dl}(313))	315.	O, H_a, H	S
316.	O, H_a, T_a	U	317.	O, H_a, T_b	U	318.	O, H_a, T_c	U (S_{dl}(317))
319.	O, H_a, I	U	320.	O, H_b, H_c	U (S_{dl}(313))	321.	O, H_b, H	S (S_d(315))
322.	O, H_b, T_a	U (S_d(317))	323.	O, H_b, T_b	U (S_d(316))	324.	O, H_b, T_c	U (S_{dl}(317))
325.	O, H_b, I	U (S_d(319))	326.	O, H_c, H	S (S_{dl}(315))	327.	O, H_c, T_a	U (S_{dl}(317))
328.	O, H_c, T_b	U (S_{dl}(317))	329.	O, H_c, T_c	U (S_{dl}(316))	330.	O, H_c, I	U (S_{dl}(319))
331.	O, H, T_a	U	332.	O, H, T_b	U (S_d(331))	333.	O, H, T_c	U (S_{dl}(331))
334.	O, H, I	U	335.	O, T_a, T_b	U	336.	O, T_a, T_c	U (S_{dl}(335))
337.	O, T_a, I	U	338.	O, T_b, T_c	U (S_{dl}(335))	339.	O, T_b, I	U (S_d(337))
340.	O, T_c, I	U (S_{dl}(337))	341.	M_a, M_b, M_c	S	342.	M_a, M_b, G	S
343.	M_a, M_b, H_a	S	344.	M_a, M_b, H_b	S (S_d(343))	345.	M_a, M_b, H_c	S
346.	M_a, M_b, H	U	347.	M_a, M_b, T_a	U	348.	M_a, M_b, T_b	U (S_d(347))
349.	M_a, M_b, T_c	U	350.	M_a, M_b, I	U	351.	M_a, M_c, G	S (S_{dl}(342))
352.	M_a, M_c, H_a	S (S_d(343))	353.	M_a, M_c, H_b	S (S_d(345))	354.	M_a, M_c, H_c	S (S_d(343))
355.	M_a, M_c, H	U (S_{dl}(346))	356.	M_a, M_c, T_a	U (S_d(347))	357.	M_a, M_c, T_b	U (S_d(349))
358.	M_a, M_c, T_c	U (S_d(347))	359.	M_a, M_c, I	U (S_{dl}(350))	360.	M_a, G, H_a	L
361.	M_a, G, H_b	S	362.	M_a, G, H_c	S (S_{dl}(361))	363.	M_a, G, H	S
364.	M_a, G, T_a	S	365.	M_a, G, T_b	U	366.	M_a, G, T_c	U (S_{dl}(365))
367.	M_a, G, I	S	368.	M_a, H_a, H_b	S	369.	M_a, H_a, H_c	S (S_d(368))

Table A1: *Status of the problems from Wernick's corpus - continued*

370.	M_a, H_a, H	L	371.	M_a, H_a, T_a	L	372.	M_a, H_a, T_b	U
373.	M_a, H_a, T_c	U (S_d(372))	374.	M_a, H_a, I	S	375.	M_a, H_b, H_c	L
376.	M_a, H_b, H	S	377.	M_a, H_b, T_a	S	378.	M_a, H_b, T_b	S
379.	M_a, H_b, T_c	U	380.	M_a, H_b, I	U	381.	M_a, H_c, H	S (S_{dl}(376))
382.	M_a, H_c, T_a	S (S_d(377))	383.	M_a, H_c, T_b	U (S_d(379))	384.	M_a, H_c, T_c	S (S_d(378))
385.	M_a, H_c, I	U (S_{dl}(380))	386.	M_a, H, T_a	U	387.	M_a, H, T_b	U
388.	M_a, H, T_c	U (S_{dl}(387))	389.	M_a, H, I	U	390.	M_a, T_a, T_b	U
391.	M_a, T_a, T_c	U (S_d(390))	392.	M_a, T_a, I	S	393.	M_a, T_b, T_c	U
394.	M_a, T_b, I	U	395.	M_a, T_c, I	U (S_{dl}(394))	396.	M_b, M_c, G	S (S_{dl}(342))
397.	M_b, M_c, H_a	S (S_d(345))	398.	M_b, M_c, H_b	S (S_d(343))	399.	M_b, M_c, H_c	S (S_d(343))
400.	M_b, M_c, H	U (S_{dl}(346))	401.	M_b, M_c, T_a	U (S_d(349))	402.	M_b, M_c, T_b	U (S_d(347))
403.	M_b, M_c, T_c	U (S_d(347))	404.	M_b, M_c, I	U (S_{dl}(350))	405.	M_b, G, H_a	S (S_d(361))
406.	M_b, G, H_b	L (S_d(360))	407.	M_b, G, H_c	S (S_{dl}(361))	408.	M_b, G, H	S (S_d(363))
409.	M_b, G, T_a	U (S_d(365))	410.	M_b, G, T_b	S (S_d(364))	411.	M_b, G, T_c	U (S_{dl}(365))
412.	M_b, G, I	S (S_d(367))	413.	M_b, H_a, H_b	S (S_d(368))	414.	M_b, H_a, H_c	L (S_d(375))
415.	M_b, H_a, H	S (S_d(376))	416.	M_b, H_a, T_a	S (S_d(378))	417.	M_b, H_a, T_b	S (S_d(377))
418.	M_b, H_a, T_c	U (S_d(379))	419.	M_b, H_a, I	U (S_d(380))	420.	M_b, H_b, H_c	S (S_d(368))
421.	M_b, H_b, H	L (S_d(370))	422.	M_b, H_b, T_a	U (S_d(372))	423.	M_b, H_b, T_b	L (S_d(371))
424.	M_b, H_b, T_c	U (S_d(372))	425.	M_b, H_b, I	S (S_d(374))	426.	M_b, H_c, H	S (S_{dl}(376))
427.	M_b, H_c, T_a	U (S_d(379))	428.	M_b, H_c, T_b	S (S_d(377))	429.	M_b, H_c, T_c	S (S_d(378))
430.	M_b, H_c, I	U (S_{dl}(380))	431.	M_b, H, T_a	U (S_d(387))	432.	M_b, H, T_b	U (S_d(386))
433.	M_b, H, T_c	U (S_{dl}(387))	434.	M_b, H, I	U (S_d(389))	435.	M_b, T_a, T_b	U (S_d(390))
436.	M_b, T_a, T_c	U (S_d(393))	437.	M_b, T_a, I	U (S_d(394))	438.	M_b, T_b, T_c	U (S_d(390))
439.	M_b, T_b, I	S (S_d(392))	440.	M_b, T_c, I	U (S_{dl}(394))	441.	M_c, G, H_a	S (S_{dl}(361))
442.	M_c, G, H_b	S (S_{dl}(361))	443.	M_c, G, H_c	L (S_{dl}(360))	444.	M_c, G, H	S (S_{dl}(363))
445.	M_c, G, T_a	U (S_{dl}(365))	446.	M_c, G, T_b	U (S_{dl}(365))	447.	M_c, G, T_c	S (S_{dl}(364))
448.	M_c, G, I	S (S_{dl}(367))	449.	M_c, H_a, H_b	L (S_d(375))	450.	M_c, H_a, H_c	S (S_d(368))
451.	M_c, H_a, H	S (S_{dl}(376))	452.	M_c, H_a, T_a	S (S_d(378))	453.	M_c, H_a, T_b	U (S_d(379))
454.	M_c, H_a, T_c	S (S_d(377))	455.	M_c, H_a, I	U (S_{dl}(380))	456.	M_c, H_b, H_c	S (S_d(368))
457.	M_c, H_b, H	S (S_{dl}(376))	458.	M_c, H_b, T_a	U (S_d(379))	459.	M_c, H_b, T_b	S (S_d(378))
460.	M_c, H_b, T_c	S (S_d(377))	461.	M_c, H_b, I	U (S_{dl}(380))	462.	M_c, H_c, H	L (S_{dl}(370))
463.	M_c, H_c, T_a	U (S_d(372))	464.	M_c, H_c, T_b	U (S_d(372))	465.	M_c, H_c, T_c	L (S_d(371))
466.	M_c, H_c, I	S (S_{dl}(374))	467.	M_c, H, T_a	U (S_{dl}(387))	468.	M_c, H, T_b	U (S_{dl}(387))
469.	M_c, H, T_c	U (S_{dl}(386))	470.	M_c, H, I	U (S_{dl}(389))	471.	M_c, T_a, T_b	U (S_d(393))
472.	M_c, T_a, T_c	U (S_d(390))	473.	M_c, T_a, I	U (S_{dl}(394))	474.	M_c, T_b, T_c	U (S_d(390))
475.	M_c, T_b, I	U (S_{dl}(394))	476.	M_c, T_c, I	S (S_{dl}(392))	477.	G, H_a, H_b	U
478.	G, H_a, H_c	U (S_{dl}(477))	479.	G, H_a, H	S	480.	G, H_a, T_a	S
481.	G, H_a, T_b	U	482.	G, H_a, T_c	U (S_{dl}(481))	483.	G, H_a, I	U
484.	G, H_b, H_c	U (S_{dl}(477))	485.	G, H_b, H	S (S_d(479))	486.	G, H_b, T_a	U (S_d(481))
487.	G, H_b, T_b	S (S_d(480))	488.	G, H_b, T_c	U (S_{dl}(481))	489.	G, H_b, I	U (S_d(483))
490.	G, H_c, H	S (S_{dl}(479))	491.	G, H_c, T_a	U (S_{dl}(481))	492.	G, H_c, T_b	U (S_{dl}(481))
493.	G, H_c, T_c	S (S_{dl}(480))	494.	G, H_c, I	U (S_{dl}(483))	495.	G, H, T_a	U
496.	G, H, T_b	U (S_d(495))	497.	G, H, T_c	U (S_{dl}(495))	498.	G, H, I	U
499.	G, T_a, T_b	U	500.	G, T_a, T_c	U (S_{dl}(499))	501.	G, T_a, I	U
502.	G, T_b, T_c	U (S_{dl}(499))	503.	G, T_b, I	U (S_d(501))	504.	G, T_c, I	U (S_{dl}(501))
505.	H_a, H_b, H_c	S	506.	H_a, H_b, H	S	507.	H_a, H_b, T_a	S
508.	H_a, H_b, T_b	U (S_d(507))	509.	H_a, H_b, T_c	U	510.	H_a, H_b, I	U
511.	H_a, H_c, H	S (S_d(506))	512.	H_a, H_c, T_a	U (S_d(507))	513.	H_a, H_c, T_b	U (S_d(509))
514.	H_a, H_c, T_c	S (S_d(507))	515.	H_a, H_c, I	U (S_{dl}(510))	516.	H_a, H, T_a	L
517.	H_a, H, T_b	U	518.	H_a, H, T_c	U (S_{dl}(517))	519.	H_a, H, I	U
520.	H_a, T_a, T_b	U	521.	H_a, T_a, T_c	U (S_d(520))	522.	H_a, T_a, I	S
523.	H_a, T_b, T_c	U	524.	H_a, T_b, I	U	525.	H_a, T_c, I	U (S_{dl}(524))
526.	H_b, H_c, H	S (S_{dl}(506))	527.	H_b, H_c, T_a	U (S_d(509))	528.	H_b, H_c, T_b	S (S_d(507))
529.	H_b, H_c, T_c	U (S_d(507))	530.	H_b, H_c, I	U (S_d(510))	531.	H_b, H, T_a	U (S_d(517))
532.	H_b, H, T_b	L (S_d(516))	533.	H_b, H, T_c	U (S_{dl}(517))	534.	H_b, H, I	U (S_d(519))
535.	H_b, T_a, T_b	U (S_d(520))	536.	H_b, T_a, T_c	U (S_d(523))	537.	H_b, T_a, I	U (S_d(524))
538.	H_b, T_b, T_c	U (S_d(520))	539.	H_b, T_b, I	S (S_d(522))	540.	H_b, T_c, I	U (S_{dl}(524))
541.	H_c, H, T_a	U (S_{dl}(517))	542.	H_c, H, T_b	U (S_{dl}(517))	543.	H_c, H, T_c	L (S_{dl}(516))
544.	H_c, H, I	U (S_{dl}(519))	545.	H_c, T_a, T_b	U (S_d(523))	546.	H_c, T_a, T_c	U (S_d(520))
547.	H_c, T_a, I	U (S_{dl}(524))	548.	H_c, T_b, T_c	U (S_d(520))	549.	H_c, T_b, I	U (S_{dl}(524))
550.	H_c, T_c, I	S (S_{dl}(522))	551.	H, T_a, T_b	U	552.	H, T_a, T_c	U (S_{dl}(551))
553.	H, T_a, I	U	554.	H, T_b, T_c	U (S_{dl}(551))	555.	H, T_b, I	U (S_d(553))
556.	H, T_c, I	U (S_{dl}(553))	557.	T_a, T_b, T_c	U	558.	T_a, T_b, I	U
559.	T_a, T_c, I	U (S_{dl}(558))	560.	T_b, T_c, I	U (S_{dl}(558))			

Table A2.: Status of the problems from Connelly's corpus (legend is the same as in previous table)

			1001.	A, B, E_a	S	1002.	A, B, E_b	S (S_d(1001))
1003.	A, B, E_c	S	1004.	A, B, N	S	1005.	A, C, E_a	S (S_d(1001))

Table A2: *Status of the problems from Connelly's corpus - continued*

1006.	A, C, E_b	S ($S_d(1003)$)	1007.	A, C, E_c	S ($S_d(1001)$)	1008.	A, C, N	S ($S_{dl}(1004)$)
1009.	A, E_a, E_b	S	1010.	A, E_a, E_c	S ($S_d(1009)$)	1011.	A, E_a, G	S
1012.	A, E_a, H	R	1013.	A, E_a, H_a	L	1014.	A, E_a, H_b	L
1015.	A, E_a, H_c	L ($S_d(1014)$)	1016.	A, E_a, I	U	1017.	A, E_a, M_a	S
1018.	A, E_a, M_b	S	1019.	A, E_a, M_c	S ($S_d(1018)$)	1020.	A, E_a, N	S
1021.	A, E_a, O	S	1022.	A, E_a, T_a	S	1023.	A, E_a, T_b	U
1024.	A, E_a, T_c	U ($S_d(1023)$)	1025.	A, E_b, E_c	U	1026.	A, E_b, G	U
1027.	A, E_b, H	S	1028.	A, E_b, H_a	S	1029.	A, E_b, H_b	L
1030.	A, E_b, H_c	S	1031.	A, E_b, I	U	1032.	A, E_b, M_a	U
1033.	A, E_b, M_b	S	1034.	A, E_b, M_c	S	1035.	A, E_b, N	S
1036.	A, E_b, O	U	1037.	A, E_b, T_a	U	1038.	A, E_b, T_b	U
1039.	A, E_b, T_c	U	1040.	A, E_c, G	U ($S_{dl}(1026)$)	1041.	A, E_c, H	S ($S_{dl}(1027)$)
1042.	A, E_c, H_a	S ($S_d(1028)$)	1043.	A, E_c, H_b	S ($S_d(1030)$)	1044.	A, E_c, H_c	L ($S_d(1029)$)
1045.	A, E_c, I	U ($S_{dl}(1031)$)	1046.	A, E_c, M_a	U ($S_d(1032)$)	1047.	A, E_c, M_b	S ($S_d(1034)$)
1048.	A, E_c, M_c	S ($S_d(1033)$)	1049.	A, E_c, N	S ($S_{dl}(1035)$)	1050.	A, E_c, O	U ($S_{dl}(1036)$)
1051.	A, E_c, T_a	U ($S_d(1037)$)	1052.	A, E_c, T_b	U ($S_d(1039)$)	1053.	A, E_c, T_c	U ($S_d(1038)$)
1054.	A, G, N	S	1055.	A, H, N	S	1056.	A, H_a, N	S
1057.	A, H_b, N	S	1058.	A, H_c, N	S ($S_{dl}(1057)$)	1059.	A, I, N	U
1060.	A, M_a, N	S	1061.	A, M_b, N	S	1062.	A, M_c, N	S ($S_{dl}(1061)$)
1063.	A, N, O	S	1064.	A, N, T_a	U	1065.	A, N, T_b	U
1066.	A, N, T_c	U ($S_{dl}(1065)$)	1067.	B, C, E_a	S ($S_d(1003)$)	1068.	B, C, E_b	S ($S_d(1001)$)
1069.	B, C, E_c	S ($S_d(1001)$)	1070.	B, C, N	S ($S_{dl}(1004)$)	1071.	B, E_a, E_b	S ($S_d(1009)$)
1072.	B, E_a, E_c	U ($S_d(1025)$)	1073.	B, E_a, G	U ($S_d(1026)$)	1074.	B, E_a, H	S ($S_d(1027)$)
1075.	B, E_a, H_a	L ($S_d(1029)$)	1076.	B, E_a, H_b	S ($S_d(1028)$)	1077.	B, E_a, H_c	S ($S_d(1030)$)
1078.	B, E_a, I	U ($S_d(1031)$)	1079.	B, E_a, M_a	S ($S_d(1033)$)	1080.	B, E_a, M_b	U ($S_d(1032)$)
1081.	B, E_a, M_c	S ($S_d(1034)$)	1082.	B, E_a, N	S ($S_d(1035)$)	1083.	B, E_a, O	U ($S_d(1036)$)
1084.	B, E_a, T_a	U ($S_d(1038)$)	1085.	B, E_a, T_b	U ($S_d(1037)$)	1086.	B, E_a, T_c	U ($S_d(1039)$)
1087.	B, E_b, E_c	S ($S_d(1009)$)	1088.	B, E_b, G	S ($S_d(1011)$)	1089.	B, E_b, H	R ($S_d(1012)$)
1090.	B, E_b, H_a	L ($S_d(1014)$)	1091.	B, E_b, H_b	L ($S_d(1013)$)	1092.	B, E_b, H_c	L ($S_d(1014)$)
1093.	B, E_b, I	U ($S_d(1016)$)	1094.	B, E_b, M_a	S ($S_d(1018)$)	1095.	B, E_b, M_b	S ($S_d(1017)$)
1096.	B, E_b, M_c	S ($S_d(1018)$)	1097.	B, E_b, N	S ($S_d(1020)$)	1098.	B, E_b, O	S ($S_d(1021)$)
1099.	B, E_b, T_a	U ($S_d(1023)$)	1100.	B, E_b, T_b	S ($S_d(1022)$)	1101.	B, E_b, T_c	U ($S_d(1023)$)
1102.	B, E_c, G	U ($S_{dl}(1026)$)	1103.	B, E_c, H	S ($S_{dl}(1027)$)	1104.	B, E_c, H_a	S ($S_d(1030)$)
1105.	B, E_c, H_b	S ($S_d(1028)$)	1106.	B, E_c, H_c	L ($S_d(1029)$)	1107.	B, E_c, I	U ($S_{dl}(1031)$)
1108.	B, E_c, M_a	S ($S_d(1034)$)	1109.	B, E_c, M_b	U ($S_d(1032)$)	1110.	B, E_c, M_c	S ($S_d(1033)$)
1111.	B, E_c, N	S ($S_{dl}(1035)$)	1112.	B, E_c, O	U ($S_{dl}(1036)$)	1113.	B, E_c, T_a	U ($S_d(1039)$)
1114.	B, E_c, T_b	U ($S_d(1037)$)	1115.	B, E_c, T_c	U ($S_d(1038)$)	1116.	B, G, N	S ($S_d(1054)$)
1117.	B, H, N	S ($S_d(1055)$)	1118.	B, H_a, N	S ($S_d(1057)$)	1119.	B, H_b, N	S ($S_d(1056)$)
1120.	B, H_c, N	S ($S_{dl}(1057)$)	1121.	B, I, N	U ($S_d(1059)$)	1122.	B, M_a, N	S ($S_d(1061)$)
1123.	B, M_b, N	S ($S_d(1060)$)	1124.	B, M_c, N	S ($S_{dl}(1061)$)	1125.	B, N, O	S ($S_d(1063)$)
1126.	B, N, T_a	U ($S_d(1065)$)	1127.	B, N, T_b	U ($S_d(1064)$)	1128.	B, N, T_c	U ($S_{dl}(1065)$)
1129.	C, E_a, E_b	U ($S_d(1025)$)	1130.	C, E_a, E_c	S ($S_d(1009)$)	1131.	C, E_a, G	U ($S_{dl}(1026)$)
1132.	C, E_a, H	S ($S_{dl}(1027)$)	1133.	C, E_a, H_a	L ($S_d(1029)$)	1134.	C, E_a, H_b	S ($S_d(1030)$)
1135.	C, E_a, H_c	S ($S_d(1028)$)	1136.	C, E_a, I	U ($S_{dl}(1031)$)	1137.	C, E_a, M_a	S ($S_d(1033)$)
1138.	C, E_a, M_b	S ($S_d(1034)$)	1139.	C, E_a, M_c	U ($S_d(1032)$)	1140.	C, E_a, N	S ($S_{dl}(1035)$)
1141.	C, E_a, O	U ($S_{dl}(1036)$)	1142.	C, E_a, T_a	U ($S_d(1038)$)	1143.	C, E_a, T_b	U ($S_d(1039)$)
1144.	C, E_a, T_c	U ($S_d(1037)$)	1145.	C, E_b, E_c	S ($S_d(1009)$)	1146.	C, E_b, G	U ($S_{dl}(1026)$)
1147.	C, E_b, H	S ($S_{dl}(1027)$)	1148.	C, E_b, H_a	S ($S_d(1030)$)	1149.	C, E_b, H_b	L ($S_d(1029)$)
1150.	C, E_b, H_c	S ($S_d(1028)$)	1151.	C, E_b, I	U ($S_{dl}(1031)$)	1152.	C, E_b, M_a	S ($S_d(1034)$)
1153.	C, E_b, M_b	S ($S_d(1033)$)	1154.	C, E_b, M_c	U ($S_d(1032)$)	1155.	C, E_b, N	S ($S_{dl}(1035)$)
1156.	C, E_b, O	U ($S_{dl}(1036)$)	1157.	C, E_b, T_a	U ($S_d(1039)$)	1158.	C, E_b, T_b	U ($S_d(1038)$)
1159.	C, E_b, T_c	U ($S_d(1037)$)	1160.	C, E_c, G	S ($S_{dl}(1011)$)	1161.	C, E_c, H	R ($S_{dl}(1012)$)
1162.	C, E_c, H_a	L ($S_d(1014)$)	1163.	C, E_c, H_b	L ($S_d(1014)$)	1164.	C, E_c, H_c	L ($S_d(1013)$)
1165.	C, E_c, I	U ($S_{dl}(1016)$)	1166.	C, E_c, M_a	S ($S_d(1018)$)	1167.	C, E_c, M_b	S ($S_d(1018)$)
1168.	C, E_c, M_c	S ($S_d(1017)$)	1169.	C, E_c, N	S ($S_{dl}(1020)$)	1170.	C, E_c, O	S ($S_{dl}(1021)$)
1171.	C, E_c, T_a	U ($S_d(1023)$)	1172.	C, E_c, T_b	U ($S_d(1023)$)	1173.	C, E_c, T_c	S ($S_d(1022)$)
1174.	C, G, N	S ($S_{dl}(1054)$)	1175.	C, H, N	S ($S_{dl}(1055)$)	1176.	C, H_a, N	S ($S_{dl}(1057)$)
1177.	C, H_b, N	S ($S_{dl}(1057)$)	1178.	C, H_c, N	S ($S_{dl}(1056)$)	1179.	C, I, N	U ($S_{dl}(1059)$)
1180.	C, M_a, N	S ($S_{dl}(1061)$)	1181.	C, M_b, N	S ($S_{dl}(1061)$)	1182.	C, M_c, N	S ($S_{dl}(1060)$)
1183.	C, N, O	S ($S_{dl}(1063)$)	1184.	C, N, T_a	U ($S_{dl}(1065)$)	1185.	C, N, T_b	U ($S_{dl}(1065)$)
1186.	C, N, T_c	U ($S_{dl}(1064)$)	1187.	E_a, E_b, E_c	S	1188.	E_a, E_b, G	U
1189.	E_a, E_b, H	S	1190.	E_a, E_b, H_a	S	1191.	E_a, E_b, H_b	S ($S_d(1190)$)
1192.	E_a, E_b, H_c	S	1193.	E_a, E_b, I	U	1194.	E_a, E_b, M_a	L
1195.	E_a, E_b, M_b	L ($S_d(1194)$)	1196.	E_a, E_b, M_c	S	1197.	E_a, E_b, N	L
1198.	E_a, E_b, O	U	1199.	E_a, E_b, T_a	U	1200.	E_a, E_b, T_b	U ($S_d(1199)$)
1201.	E_a, E_b, T_c	U	1202.	E_a, E_c, G	U ($S_{dl}(1188)$)	1203.	E_a, E_c, H	S ($S_{dl}(1189)$)
1204.	E_a, E_c, H_a	S ($S_d(1190)$)	1205.	E_a, E_c, H_b	S ($S_d(1192)$)	1206.	E_a, E_c, H_c	S ($S_d(1190)$)
1207.	E_a, E_c, I	U ($S_{dl}(1193)$)	1208.	E_a, E_c, M_a	L ($S_d(1194)$)	1209.	E_a, E_c, M_b	S ($S_d(1196)$)
1210.	E_a, E_c, M_c	L ($S_d(1194)$)	1211.	E_a, E_c, N	L ($S_{dl}(1197)$)	1212.	E_a, E_c, O	U ($S_{dl}(1198)$)
1213.	E_a, E_c, T_a	U ($S_d(1199)$)	1214.	E_a, E_c, T_b	U ($S_d(1201)$)	1215.	E_a, E_c, T_c	U ($S_d(1199)$)
1216.	E_a, G, H	S	1217.	E_a, G, H_a	S	1218.	E_a, G, H_b	U
1219.	E_a, G, H_c	U ($S_{dl}(1218)$)	1220.	E_a, G, I	U	1221.	E_a, G, M_a	S
1222.	E_a, G, M_b	U	1223.	E_a, G, M_c	U ($S_{dl}(1222)$)	1224.	E_a, G, N	S
1225.	E_a, G, O	S	1226.	E_a, G, T_a	U	1227.	E_a, G, T_b	U
1228.	E_a, G, T_c	U ($S_{dl}(1227)$)	1229.	E_a, H, H_a	L	1230.	E_a, H, H_b	L

Table A2: *Status of the problems from Connelly's corpus - continued*

1231.	E_a, H, H_c	L ($S_{dl}(1230)$)	1232.	E_a, H, I	U	1233.	E_a, H, M_a	S
1234.	E_a, H, M_b	S	1235.	E_a, H, M_c	S ($S_{dl}(1234)$)	1236.	E_a, H, N	S
1237.	E_a, H, O	S	1238.	E_a, H, T_a	S	1239.	E_a, H, T_b	U
1240.	E_a, H, T_c	U ($S_{dl}(1239)$)	1241.	E_a, H_a, H_b	S	1242.	E_a, H_a, H_c	S ($S_d(1241)$)
1243.	E_a, H_a, I	U	1244.	E_a, H_a, M_a	L	1245.	E_a, H_a, M_b	S
1246.	E_a, H_a, M_c	S ($S_d(1245)$)	1247.	E_a, H_a, N	L	1248.	E_a, H_a, O	S
1249.	E_a, H_a, T_a	L	1250.	E_a, H_a, T_b	U	1251.	E_a, H_a, T_c	U ($S_d(1250)$)
1252.	E_a, H_b, H_c	L	1253.	E_a, H_b, I	U	1254.	E_a, H_b, M_a	L
1255.	E_a, H_b, M_b	S	1256.	E_a, H_b, M_c	S	1257.	E_a, H_b, N	L
1258.	E_a, H_b, O	U	1259.	E_a, H_b, T_a	U	1260.	E_a, H_b, T_b	U
1261.	E_a, H_b, T_c	U	1262.	E_a, H_c, I	U ($S_{dl}(1253)$)	1263.	E_a, H_c, M_a	L ($S_d(1254)$)
1264.	E_a, H_c, M_b	S ($S_d(1256)$)	1265.	E_a, H_c, M_c	S ($S_d(1255)$)	1266.	E_a, H_c, N	L ($S_{dl}(1257)$)
1267.	E_a, H_c, O	U ($S_{dl}(1258)$)	1268.	E_a, H_c, T_a	U ($S_d(1259)$)	1269.	E_a, H_c, T_b	U ($S_d(1261)$)
1270.	E_a, H_c, T_c	U ($S_d(1260)$)	1271.	E_a, I, M_a	S	1272.	E_a, I, M_b	U
1273.	E_a, I, M_c	U ($S_{dl}(1272)$)	1274.	E_a, I, N	S	1275.	E_a, I, O	U
1276.	E_a, I, T_a	U	1277.	E_a, I, T_b	U	1278.	E_a, I, T_c	U ($S_{dl}(1277)$)
1279.	E_a, M_a, M_b	L	1280.	E_a, M_a, M_c	L ($S_d(1279)$)	1281.	E_a, M_a, N	R
1282.	E_a, M_a, O	S	1283.	E_a, M_a, T_a	U	1284.	E_a, M_a, T_b	U
1285.	E_a, M_a, T_c	U ($S_d(1284)$)	1286.	E_a, M_b, M_c	S	1287.	E_a, M_b, N	L
1288.	E_a, M_b, O	S	1289.	E_a, M_b, T_a	U	1290.	E_a, M_b, T_b	U
1291.	E_a, M_b, T_c	U	1292.	E_a, M_c, N	L ($S_{dl}(1287)$)	1293.	E_a, M_c, O	S ($S_{dl}(1288)$)
1294.	E_a, M_c, T_a	U ($S_d(1289)$)	1295.	E_a, M_c, T_b	U ($S_d(1291)$)	1296.	E_a, M_c, T_c	U ($S_d(1290)$)
1297.	E_a, N, O	S	1298.	E_a, N, T_a	U	1299.	E_a, N, T_b	U
1300.	E_a, N, T_c	U ($S_{dl}(1299)$)	1301.	E_a, O, T_a	U	1302.	E_a, O, T_b	U
1303.	E_a, O, T_c	U ($S_{dl}(1302)$)	1304.	E_a, T_a, T_b	U	1305.	E_a, T_a, T_c	U ($S_d(1304)$)
1306.	E_a, T_b, T_c	U	1307.	E_b, E_c, G	U ($S_{dl}(1188)$)	1308.	E_b, E_c, H	S ($S_{dl}(1189)$)
1309.	E_b, E_c, H_a	S ($S_d(1192)$)	1310.	E_b, E_c, H_b	S ($S_d(1190)$)	1311.	E_b, E_c, H_c	S ($S_d(1190)$)
1312.	E_b, E_c, I	U ($S_{dl}(1193)$)	1313.	E_b, E_c, M_a	S ($S_d(1196)$)	1314.	E_b, E_c, M_b	L ($S_d(1194)$)
1315.	E_b, E_c, M_c	L ($S_d(1194)$)	1316.	E_b, E_c, N	L ($S_{dl}(1197)$)	1317.	E_b, E_c, O	U ($S_{dl}(1198)$)
1318.	E_b, E_c, T_a	U ($S_d(1201)$)	1319.	E_b, E_c, T_b	U ($S_d(1199)$)	1320.	E_b, E_c, T_c	U ($S_d(1199)$)
1321.	E_b, G, H	S ($S_d(1216)$)	1322.	E_b, G, H_a	U ($S_d(1218)$)	1323.	E_b, G, H_b	S ($S_d(1217)$)
1324.	E_b, G, H_c	U ($S_{dl}(1218)$)	1325.	E_b, G, I	U ($S_d(1220)$)	1326.	E_b, G, M_a	U ($S_d(1222)$)
1327.	E_b, G, M_b	S ($S_d(1221)$)	1328.	E_b, G, M_c	U ($S_{dl}(1222)$)	1329.	E_b, G, N	S ($S_d(1224)$)
1330.	E_b, G, O	S ($S_d(1225)$)	1331.	E_b, G, T_a	U ($S_d(1227)$)	1332.	E_b, G, T_b	U ($S_d(1226)$)
1333.	E_b, G, T_c	U ($S_{dl}(1227)$)	1334.	E_b, H, H_a	L ($S_d(1230)$)	1335.	E_b, H, H_b	L ($S_d(1229)$)
1336.	E_b, H, H_c	L ($S_{dl}(1230)$)	1337.	E_b, H, I	U ($S_d(1232)$)	1338.	E_b, H, M_a	S ($S_d(1234)$)
1339.	E_b, H, M_b	S ($S_d(1233)$)	1340.	E_b, H, M_c	S ($S_{dl}(1234)$)	1341.	E_b, H, N	S ($S_d(1236)$)
1342.	E_b, H, O	S ($S_d(1237)$)	1343.	E_b, H, T_a	U ($S_d(1239)$)	1344.	E_b, H, T_b	S ($S_d(1238)$)
1345.	E_b, H, T_c	U ($S_{dl}(1239)$)	1346.	E_b, H_a, H_b	S ($S_d(1241)$)	1347.	E_b, H_a, H_c	L ($S_d(1252)$)
1348.	E_b, H_a, I	U ($S_d(1253)$)	1349.	E_b, H_a, M_a	S ($S_d(1255)$)	1350.	E_b, H_a, M_b	L ($S_d(1254)$)
1351.	E_b, H_a, M_c	S ($S_d(1256)$)	1352.	E_b, H_a, N	L ($S_d(1257)$)	1353.	E_b, H_a, O	U ($S_d(1258)$)
1354.	E_b, H_a, T_a	U ($S_d(1260)$)	1355.	E_b, H_a, T_b	U ($S_d(1259)$)	1356.	E_b, H_a, T_c	U ($S_d(1261)$)
1357.	E_b, H_b, H_c	S ($S_d(1241)$)	1358.	E_b, H_b, I	U ($S_d(1243)$)	1359.	E_b, H_b, M_a	S ($S_d(1245)$)
1360.	E_b, H_b, M_b	L ($S_d(1244)$)	1361.	E_b, H_b, M_c	S ($S_d(1245)$)	1362.	E_b, H_b, N	L ($S_d(1247)$)
1363.	E_b, H_b, O	S ($S_d(1248)$)	1364.	E_b, H_b, T_a	U ($S_d(1250)$)	1365.	E_b, H_b, T_b	L ($S_d(1249)$)
1366.	E_b, H_b, T_c	U ($S_d(1250)$)	1367.	E_b, H_c, I	U ($S_{dl}(1253)$)	1368.	E_b, H_c, M_a	S ($S_d(1256)$)
1369.	E_b, H_c, M_b	L ($S_d(1254)$)	1370.	E_b, H_c, M_c	S ($S_d(1255)$)	1371.	E_b, H_c, N	L ($S_{dl}(1257)$)
1372.	E_b, H_c, O	U ($S_{dl}(1258)$)	1373.	E_b, H_c, T_a	U ($S_d(1261)$)	1374.	E_b, H_c, T_b	U ($S_d(1259)$)
1375.	E_b, H_c, T_c	U ($S_d(1260)$)	1376.	E_b, I, M_a	U ($S_d(1272)$)	1377.	E_b, I, M_b	S ($S_d(1271)$)
1378.	E_b, I, M_c	U ($S_{dl}(1272)$)	1379.	E_b, I, N	S ($S_d(1274)$)	1380.	E_b, I, O	U ($S_d(1275)$)
1381.	E_b, I, T_a	U ($S_d(1277)$)	1382.	E_b, I, T_b	U ($S_d(1276)$)	1383.	E_b, I, T_c	U ($S_{dl}(1277)$)
1384.	E_b, M_a, M_b	L ($S_d(1279)$)	1385.	E_b, M_a, M_c	S ($S_d(1286)$)	1386.	E_b, M_a, N	L ($S_d(1287)$)
1387.	E_b, M_a, O	S ($S_d(1288)$)	1388.	E_b, M_a, T_a	U ($S_d(1290)$)	1389.	E_b, M_a, T_b	U ($S_d(1289)$)
1390.	E_b, M_a, T_c	U ($S_d(1291)$)	1391.	E_b, M_b, M_c	L ($S_d(1279)$)	1392.	E_b, M_b, N	R ($S_d(1281)$)
1393.	E_b, M_b, O	S ($S_d(1282)$)	1394.	E_b, M_b, T_a	U ($S_d(1284)$)	1395.	E_b, M_b, T_b	U ($S_d(1283)$)
1396.	E_b, M_b, T_c	U ($S_d(1284)$)	1397.	E_b, M_c, N	L ($S_{dl}(1287)$)	1398.	E_b, M_c, O	S ($S_{dl}(1288)$)
1399.	E_b, M_c, T_a	U ($S_d(1291)$)	1400.	E_b, M_c, T_b	U ($S_d(1289)$)	1401.	E_b, M_c, T_c	U ($S_d(1290)$)
1402.	E_b, N, O	S ($S_d(1297)$)	1403.	E_b, N, T_a	U ($S_d(1299)$)	1404.	E_b, N, T_b	U ($S_d(1298)$)
1405.	E_b, N, T_c	U ($S_{dl}(1299)$)	1406.	E_b, O, T_a	U ($S_d(1302)$)	1407.	E_b, O, T_b	U ($S_d(1301)$)
1408.	E_b, O, T_c	U ($S_{dl}(1302)$)	1409.	E_b, T_a, T_b	U ($S_d(1304)$)	1410.	E_b, T_a, T_c	U ($S_d(1306)$)
1411.	E_b, T_b, T_c	U ($S_d(1304)$)	1412.	E_c, G, H	S ($S_{dl}(1216)$)	1413.	E_c, G, H_a	U ($S_{dl}(1218)$)
1414.	E_c, G, H_b	U ($S_{dl}(1218)$)	1415.	E_c, G, H_c	S ($S_{dl}(1217)$)	1416.	E_c, G, I	U ($S_{dl}(1220)$)
1417.	E_c, G, M_a	U ($S_{dl}(1222)$)	1418.	E_c, G, M_b	U ($S_{dl}(1222)$)	1419.	E_c, G, M_c	S ($S_{dl}(1221)$)
1420.	E_c, G, N	S ($S_{dl}(1224)$)	1421.	E_c, G, O	S ($S_{dl}(1225)$)	1422.	E_c, G, T_a	U ($S_{dl}(1227)$)
1423.	E_c, G, T_b	U ($S_{dl}(1227)$)	1424.	E_c, G, T_c	U ($S_{dl}(1226)$)	1425.	E_c, H, H_a	L ($S_{dl}(1230)$)
1426.	E_c, H, H_b	L ($S_{dl}(1230)$)	1427.	E_c, H, H_c	L ($S_{dl}(1229)$)	1428.	E_c, H, I	U ($S_{dl}(1232)$)
1429.	E_c, H, M_a	S ($S_{dl}(1234)$)	1430.	E_c, H, M_b	S ($S_{dl}(1234)$)	1431.	E_c, H, M_c	S ($S_{dl}(1233)$)
1432.	E_c, H, N	S ($S_{dl}(1236)$)	1433.	E_c, H, O	S ($S_{dl}(1237)$)	1434.	E_c, H, T_a	U ($S_{dl}(1239)$)
1435.	E_c, H, T_b	U ($S_{dl}(1239)$)	1436.	E_c, H, T_c	S ($S_{dl}(1238)$)	1437.	E_c, H_a, H_b	L ($S_d(1252)$)
1438.	E_c, H_a, H_c	S ($S_d(1241)$)	1439.	E_c, H_a, I	U ($S_{dl}(1253)$)	1440.	E_c, H_a, M_a	S ($S_d(1255)$)
1441.	E_c, H_a, M_b	S ($S_d(1256)$)	1442.	E_c, H_a, M_c	L ($S_d(1254)$)	1443.	E_c, H_a, N	L ($S_{dl}(1257)$)
1444.	E_c, H_a, O	U ($S_{dl}(1258)$)	1445.	E_c, H_a, T_a	U ($S_d(1260)$)	1446.	E_c, H_a, T_b	U ($S_d(1261)$)
1447.	E_c, H_a, T_c	U ($S_d(1259)$)	1448.	E_c, H_b, H_c	S ($S_d(1241)$)	1449.	E_c, H_b, I	U ($S_{dl}(1253)$)
1450.	E_c, H_b, M_a	S ($S_d(1256)$)	1451.	E_c, H_b, M_b	S ($S_d(1255)$)	1452.	E_c, H_b, M_c	L ($S_d(1254)$)
1453.	E_c, H_b, N	L ($S_{dl}(1257)$)	1454.	E_c, H_b, O	U ($S_{dl}(1258)$)	1455.	E_c, H_b, T_a	U ($S_d(1261)$)

Table A2: *Status of the problems from Connelly's corpus - continued*

1456.	E_c, H_b, T_b	U (S_d(1260))	1457.	E_c, H_b, T_c	U (S_d(1259))	1458.	E_c, H_c, I	U (S_{dl}(1243))
1459.	E_c, H_c, M_a	S (S_d(1245))	1460.	E_c, H_c, M_b	S (S_d(1245))	1461.	E_c, H_c, M_c	L (S_d(1244))
1462.	E_c, H_c, N	L (S_{dl}(1247))	1463.	E_c, H_c, O	S (S_{dl}(1248))	1464.	E_c, H_c, T_a	U (S_d(1250))
1465.	E_c, H_c, T_b	U (S_d(1250))	1466.	E_c, H_c, T_c	L (S_d(1249))	1467.	E_c, I, M_a	U (S_{dl}(1272))
1468.	E_c, I, M_b	U (S_{dl}(1272))	1469.	E_c, I, M_c	S (S_{dl}(1271))	1470.	E_c, I, N	S (S_{dl}(1274))
1471.	E_c, I, O	U (S_{dl}(1275))	1472.	E_c, I, T_a	U (S_{dl}(1277))	1473.	E_c, I, T_b	U (S_{dl}(1277))
1474.	E_c, I, T_c	U (S_{dl}(1276))	1475.	E_c, M_a, M_b	S (S_d(1286))	1476.	E_c, M_a, M_c	L (S_d(1279))
1477.	E_c, M_a, N	L (S_{dl}(1287))	1478.	E_c, M_a, O	S (S_{dl}(1288))	1479.	E_c, M_a, T_a	U (S_d(1290))
1480.	E_c, M_a, T_b	U (S_d(1291))	1481.	E_c, M_a, T_c	U (S_d(1289))	1482.	E_c, M_b, M_c	L (S_d(1279))
1483.	E_c, M_b, N	L (S_{dl}(1287))	1484.	E_c, M_b, O	S (S_{dl}(1288))	1485.	E_c, M_b, T_a	U (S_d(1291))
1486.	E_c, M_b, T_b	U (S_d(1290))	1487.	E_c, M_b, T_c	U (S_d(1289))	1488.	E_c, M_c, N	R (S_{dl}(1281))
1489.	E_c, M_c, O	S (S_{dl}(1282))	1490.	E_c, M_c, T_a	U (S_d(1284))	1491.	E_c, M_c, T_b	U (S_d(1284))
1492.	E_c, M_c, T_c	U (S_d(1283))	1493.	E_c, N, O	S (S_{dl}(1297))	1494.	E_c, N, T_a	U (S_{dl}(1299))
1495.	E_c, N, T_b	U (S_{dl}(1299))	1496.	E_c, N, T_c	U (S_{dl}(1298))	1497.	E_c, O, T_a	U (S_{dl}(1302))
1498.	E_c, O, T_b	U (S_{dl}(1302))	1499.	E_c, O, T_c	U (S_{dl}(1301))	1500.	E_c, T_a, T_b	U (S_d(1306))
1501.	E_c, T_a, T_c	U (S_d(1304))	1502.	E_c, T_b, T_c	U (S_d(1304))	1503.	G, H, N	R
1504.	G, H_a, N	S	1505.	G, H_b, N	S (S_d(1504))	1506.	G, H_c, N	S (S_{dl}(1504))
1507.	G, I, N	U	1508.	G, M_a, N	S	1509.	G, M_b, N	S (S_d(1508))
1510.	G, M_c, N	S (S_{dl}(1508))	1511.	G, N, O	R	1512.	G, N, T_a	U
1513.	G, N, T_b	U (S_d(1512))	1514.	G, N, T_c	U (S_{dl}(1512))	1515.	H, H_a, N	S
1516.	H, H_b, N	S (S_d(1515))	1517.	H, H_c, N	S (S_{dl}(1515))	1518.	H, I, N	U
1519.	H, M_a, N	S	1520.	H, M_b, N	S (S_d(1519))	1521.	H, M_c, N	S (S_{dl}(1519))
1522.	H, N, O	R	1523.	H, N, T_a	U	1524.	H, N, T_b	U (S_d(1523))
1525.	H, N, T_c	U (S_{dl}(1523))	1526.	H_a, H_b, N	L	1527.	H_a, H_c, N	L (S_{dl}(1526))
1528.	H_a, I, N	S	1529.	H_a, M_a, N	L	1530.	H_a, M_b, N	L
1531.	H_a, M_c, N	L (S_{dl}(1530))	1532.	H_a, N, O	S	1533.	H_a, N, T_a	U
1534.	H_a, N, T_b	U	1535.	H_a, N, T_c	U (S_{dl}(1534))	1536.	H_b, H_c, N	L (S_{dl}(1526))
1537.	H_b, I, N	S (S_d(1528))	1538.	H_b, M_a, N	L (S_d(1530))	1539.	H_b, M_b, N	L (S_d(1529))
1540.	H_b, M_c, N	L (S_{dl}(1530))	1541.	H_b, N, O	S (S_d(1532))	1542.	H_b, N, T_a	U (S_d(1534))
1543.	H_b, N, T_b	U (S_d(1533))	1544.	H_b, N, T_c	U (S_{dl}(1534))	1545.	H_c, I, N	S (S_{dl}(1528))
1546.	H_c, M_a, N	L (S_{dl}(1530))	1547.	H_c, M_b, N	L (S_{dl}(1530))	1548.	H_c, M_c, N	L (S_{dl}(1529))
1549.	H_c, N, O	S (S_{dl}(1532))	1550.	H_c, N, T_a	U (S_{dl}(1534))	1551.	H_c, N, T_b	U (S_{dl}(1534))
1552.	H_c, N, T_c	U (S_{dl}(1533))	1553.	I, M_a, N	S	1554.	I, M_b, N	S (S_d(1553))
1555.	I, M_c, N	S (S_{dl}(1553))	1556.	I, N, O	U	1557.	I, N, T_a	U
1558.	I, N, T_b	U (S_d(1557))	1559.	I, N, T_c	U (S_{dl}(1557))	1560.	M_a, M_b, N	L
1561.	M_a, M_c, N	L (S_{dl}(1560))	1562.	M_a, N, O	S	1563.	M_a, N, T_a	U
1564.	M_a, N, T_b	U	1565.	M_a, N, T_c	U (S_{dl}(1564))	1566.	M_b, M_c, N	L (S_{dl}(1560))
1567.	M_b, N, O	S (S_d(1562))	1568.	M_b, N, T_a	U (S_d(1564))	1569.	M_b, N, T_b	U (S_d(1563))
1570.	M_b, N, T_c	U (S_{dl}(1564))	1571.	M_c, N, O	S (S_{dl}(1562))	1572.	M_c, N, T_a	U (S_{dl}(1564))
1573.	M_c, N, T_b	U (S_{dl}(1564))	1574.	M_c, N, T_c	U (S_{dl}(1563))	1575.	N, O, T_a	U
1576.	N, O, T_b	U (S_d(1575))	1577.	N, O, T_c	U (S_{dl}(1575))	1578.	N, T_a, T_b	U
1579.	N, T_a, T_c	U (S_{dl}(1578))	1580.	N, T_b, T_c	U (S_{dl}(1578))			

Appendix B. Lists of definitions, lemmas, and primitive constructions used

B.1. *Wernick's corpus*

Instantiated definitions:

(1) O (circumcenter): intersection point of the bisectors of the segments BC and AC;
(2) I (incenter): intersection point of the bisectors of internal angles AT_a and BT_b;
(3) H (orthocenter): intersection point of the altitudes AH_a and BH_b;
(4) G (centroid): intersection point of medians AM_a and BM_b;
(5) H_a, H_b, H_c: intersection points of the altitude with the opposite side of the triangle;
(6) M_a, M_b, M_c (segment midpoints): points for which it holds: $\overrightarrow{BM_a}/\overrightarrow{BC} = 1/2$, $\overrightarrow{CM_b}/\overrightarrow{CA} = 1/2$, $\overrightarrow{AM_c}/\overrightarrow{AB} = 1/2$;
(7) T_a, T_b, T_c: intersection points of the internal angle bisector with the opposite side of the triangle;
(8) N_a, N_b, N_c: intersection points of the circumcenter with internal angle bisectors AT_a, BT_b, and CT_c, respectively;
(9) T'_a, T'_b, T'_c: intersection points of the external angle bisectors with the opposite sides

of the triangle;

(10) P_a, P_b, P_c: feet from the incenter I on the sides BC, AC, and AB, respectively;

(11) P'_a, P'_b, P'_c: feet from the centers of excircles on the sides BC, AC, and AB.

General definitions:

(1) XY denotes the line which passes through points X and Y;

(2) $k(X, Y)$ denotes the circle with center at point X which passes through point Y.

Instantiated lemmas:

(1) the point O belongs to the bisector of the segment AB;

(2) the point I belongs to the segment CT_c;

(3) the point H belongs to the segment CH_c;

(4) the point G belongs to the segment CM_c;

(5) points A, B, and C belong to the angle bisectors at the vertices A, B, and C;

(6) the point T_a is at the same distance from lines AB and AC; the point T_b is at same distance from lines AB and BC; point T_c is at the same distance from lines AC and BC;

(7) points A and B belong to the circle $k(O, C)$;

(8) points P_b and P_c belong to the circle $k(I, P_a)$;

(9) points N_a, N_b, and N_c belong to the lines OM_a, OM_b, and OM_c, respectively;

(10) lines AB, BC, and CA touch the circle $k(I, P_a)$;

(11) points B and I belong to the circle $k(N_a, C)$; points C and I belong to the circle $k(N_b, A)$; points A and I belong to the circle $k(N_c, B)$;

(12) points C, H_b, and H_c belong to the circle $k(M_a, B)$; points A, H_a, and H_c belong to the circle $k(M_b, C)$; points B, H_a, and H_b belong to the circle $k(M_c, A)$;

(13) $\overrightarrow{AG}/\overrightarrow{AM_a} = 2/3$, $\overrightarrow{BG}/\overrightarrow{BM_b} = 2/3$, $\overrightarrow{CG}/\overrightarrow{CM_c} = 2/3$;

(14) $\overrightarrow{HG}/\overrightarrow{HO} = 2/3$;

(15) sides of the triangle are perpendicular to the circle $k(I, P_a)$;

(16) $\overrightarrow{P_aM_a}/\overrightarrow{P_aP'_a} = 1/2$, $\overrightarrow{P_bM_b}/\overrightarrow{P_bP'_b} = 1/2$, $\overrightarrow{P_cM_c}/\overrightarrow{P_cP'_c} = 1/2$;

(17) lines IM_a and AP'_a are parallel; lines IM_b and BP'_b are parallel; lines IM_c and CP'_c are parallel;

(18) lines M_aM_b and AB are parallel; lines M_aM_c and AC are parallel; lines M_bM_c and BC are parallel;

(19) the point A belongs to the circle with diameter $T_aT'_a$; the point B belongs to the circle with diameter $T_bT'_b$; the point C belongs to the circle with diameter $T_cT'_c$;

(20) the point P_a belongs to the circle with diameter IM_a; the point P_b belongs to the circle with diameter IM_b; the point P_c belongs to the circle with diameter IM_c;

(21) $\mathcal{H}(B, C; T_a, T'_a)$, $\mathcal{H}(A, C; T_b, T'_b)$, $\mathcal{H}(A, B; T_c, T'_c)$;

(22) the point I belongs to the locus of points from which the segment T_bT_c is seen at the angle $\angle T_bIT_c$, also to the locus of points from which the segment T_bT_a is seen at the angle $\angle T_aIT_b$, as well as to the locus of points from which the segment T_aT_c is seen at the angle $\angle T_cIT_a$;

(23) $\angle BAI = \angle BAC/2$, $\angle IAC = \angle BAC/2$, $\angle CBI = \angle CBA/2$, $\angle IBA = \angle CBA/2$, $\angle ACI = \angle ACB/2$, $\angle ICB = \angle ACB/2$;

(24) $\angle HAI = \angle IAO$, $\angle HBI = \angle IBO$, $\angle HCI = \angle ICO$;

(25) $\angle H_bH_aC = \angle BAC$, $\angle H_aH_bC = \angle CBA$, $\angle H_bH_cA = \angle ACB$, $\angle H_cH_bA = \angle CBA$, $\angle H_cH_aB = \angle CBA$, $\angle H_aH_cB = \angle ACB$;

(26) $\angle T_cIT_b = \angle BAC/2 + \pi/2$, $\angle T_bIT_a = \angle ACB/2 + \pi/2$, $\angle T_aIT_c = \angle CBA/2 + \pi/2$;

(27) lines AH, BH, and CH are internal angle bisectors of the triangle $H_aH_bH_c$.

General lemmas:

(1) Center of an arbitrary circle belongs to the bisector of an arbitrary chord of that circle;
(2) If it holds $\overrightarrow{XY}/\overrightarrow{XZ} = r$, the point Z belongs to a line p, and the point Y does not belong to the same line, then the point X belongs to the line which is the image of the line p in homothety with center at point Y with coefficient $r/(r-1)$;
(3) If $\overrightarrow{XY}/\overrightarrow{ZW} = r$ holds, then $\overrightarrow{ZW}/\overrightarrow{XY} = 1/r$ holds;
(4) If $\overrightarrow{XY}/\overrightarrow{XW} = r$ holds, then $\overrightarrow{WY}/\overrightarrow{WX} = 1-r$ holds;
(5) If $\mathcal{H}(X,Y;Z,W)$ holds, then $\mathcal{H}(Y,X;W,Z)$ holds;
(6) If $\mathcal{H}(X,Y;Z,W)$ holds, then $\mathcal{H}(Z,W;X,Y)$ holds;
(7) If $\mathcal{H}(X,Y;Z,W)$ holds, then $\mathcal{H}(W,Z;Y,X)$ holds;
(8) If $\overrightarrow{XY}/\overrightarrow{XZ} = r$ and $\overrightarrow{XU}/\overrightarrow{XV} = r$ hold, for distinct points Y and U, points Y and U belong to a line p, and points Z and V belong to a line q, then lines p and q are parallel;
(9) If $\overrightarrow{XY}/\overrightarrow{XZ} = r$ holds, and points X and Z belong to a line p, then the point Y belongs to the line p as well;
(10) If a point X is at equal distance from lines p and q then lines p and q are tangents on the circle centered at point X with a radius equal to this distance;
(11) If $\alpha = \beta/2^{L_1}$ and $\gamma = K_1{\cdot}\beta/2^{K_2}+K_3{\cdot}\pi/2^{K_4}$ hold, then $\gamma = K_1{\cdot}\alpha/2^{K_2-L_1}+K_3{\cdot}\pi/2^{K_4}$ holds;
(12) If $\alpha = \beta/2^{L_1}$ and $\beta = K_1 \cdot \gamma/2^{K_2} + K_3 \cdot \pi/2^{K_4}$ hold, then $\alpha = K_1 \cdot \gamma/2^{K_2+L_1} + K_3 \cdot \pi/2^{K_4+L_1}$ holds;
(13) If $\alpha = \beta/2^{L_1}$ and $\gamma = K_1{\cdot}\alpha/2^{K_2}+K_3{\cdot}\pi/2^{K_4}$ hold, then $\gamma = K_1{\cdot}\beta/2^{K_2+L_1}+K_3{\cdot}\pi/2^{K_4}$ holds;
(14) If $\alpha = \beta/2^{L_1}$ and $\alpha = K_1 \cdot \gamma/2^{K_2} + K_3 \cdot \pi/2^{K_4}$ hold, then $\beta = K_1 \cdot \gamma/2^{K_2-L_1} + K_3 \cdot \pi/2^{K_4-L_1}$ holds;
(15) If $\overrightarrow{XY}/\overrightarrow{XZ} = r$ holds, the point X belongs to a circle C, and the point Z does not, then the point Y belongs to the image of the circle C in homothety with a center at the point Z and coefficient $r/(r-1)$;
(16) If $\alpha = \beta$ holds, then $\beta = \alpha$ holds;
(17) If $\alpha = \beta/2$ holds, then $\beta = 2 \cdot \alpha$ holds;
(18) If $\alpha = \beta/2 + \pi$ holds, then $\beta = 2 \cdot \alpha - 2 \cdot \pi$ holds.

Primitive constructions:

(1) Given points X, Z, and W, and a rational number r one can construct a point Y for which holds: $\overrightarrow{XY}/\overrightarrow{ZW} = r$; NDG condition is that the points Z and W are distinct;
(2) Given points X and Y one can construct a line XY; DET condition is that the points X and Y are distinct;
(3) Given two lines, it is possible to construct their intersection point; NDG condition is that lines are not parallel, while DET condition is that lines are not equal;
(4) Given a line and a circle, it is possible to construct their intersection points; NDG condition is that they do intersect;
(5) Given a line, a circle, and one intersection point, it is possible to construct their second intersection point; NDG condition is that the line and the circle intersect;
(6) Given two distinct points X and Y it is possible to construct a circle $k(X,Y)$ centered at point X which passes through the point Y; NDG condition is that the points X and Y are distinct;

(7) Given two circles, one can construct their intersection points; NDG condition is that the circles intersect, while DET condition is that the circles are distinct;
(8) Given two circles and one intersection point, one can construct their second intersection point; NDG condition is that the circles intersect, while DET condition is that the circles are distinct;
(9) Given points X and Y one can construct a circle with diameter $\overline{XY}$; NDG condition is that points are distinct;
(10) Given a point X and a line p one can construct a line q which passes through the point X and which is perpendicular to the line p;
(11) Given a line p and a point X which does not belong to the line p one can construct a circle k centered at point X which touches the line p; NDG condition is that the point X does not belong to the line p;
(12) Given a circle k and a point X outside the circle k, one can construct two tangents from the point X to the circle k; NDG condition is that the point X is outside the circle k;
(13) Given a circle k, a point X outside the circle k, and one tangent from the point X to the circle k, one can construct the second tangent from the point X to the circle k; NDG condition is that the point X is outside the circle k;
(14) Given points X and Y one can construct bisector of the segment $\overline{XY}$; NDG condition is that points X and Y are distinct;
(15) Given a point X, a line p and a rational number r, one can construct a line which is an image of the line p in homothety with center at the point X with coefficient r;
(16) Given a point X and a line p one can construct a line which passes through the point X and which is parallel to the line p;
(17) Given points X and Y and an angle α one can construct the line q such that that $\angle(\overline{XY}, q) = A \cdot \alpha/2^B + C \cdot \pi/2^D$ holds;[8]
(18) Given points X and Y and an angle α one can construct a line q such that $\angle(q, \overline{XY}) = A \cdot \alpha/2^B + C \cdot \pi/2^D$ holds;
(19) Given points X, Y, and Z one can construct a point W which is harmonic conjugate of Z with respect to X and Y; NDG conditions are that points X and Y are distinct, points Y and Z are distinct and point Y is not a midpoint of the segment XZ;
(20) Given points X and Y and an angle α one can construct a locus of points such that the segment $\overline{XY}$ can be seen at angle $A \cdot \alpha/2^B + C \cdot \pi/2^D$.

B.2. *Connelly's corpus*[9]

Definitions:

(1) E_a, E_b, E_c (Euler points): points for which holds: $\overrightarrow{AE_a}/\overrightarrow{AH} = 1/2$, $\overrightarrow{BE_b}/\overrightarrow{BH} = 1/2$, $\overrightarrow{CH_c}/\overrightarrow{CH} = 1/2$;
(2) N (center of the nine-point circle): circle for which holds $\overrightarrow{HN}/\overrightarrow{HO} = 1/2$;
(3) P_a^1, P_b^1, P_c^1: intersection points of the altitudes with circles centered at corresponding Euler point passing through the midpoint of opposite side of the triangle.

Lemmas:

[8] Here we use the fact that if an angle α is constructible, then the angle $K_1 \cdot \alpha/2^{K_2} + K_3 \cdot \pi/2^{K_4}$ is constructible as well, where K_1, K_2, K_3, and K_4 are natural numbers

[9] Additional knowledge needed for solving problems from Connelly's corpus is given

(1) $\overrightarrow{NG}/\overrightarrow{NO} = 1/3$; $\overrightarrow{HN}/\overrightarrow{HG} = 3/4$;
(2) points M_b, M_c, H_a, H_b, H_c, E_a, E_b, and E_c belong to the circle $k(N, M_a)$;
(3) points H, H_b, and H_c belong to the circle $k(E_a, A)$; points H, H_a, and H_c belong to the circle $k(E_b, B)$; points H, H_b, and H_a belong to the circle $k(E_c, C)$;
(4) the bisector of the angle $\angle BAC$ is parallel to the line $M_aP_a^1$; the bisector of the angle $\angle ACB$ is parallel to the line $M_cP_c^1$; the bisector of the angle $\angle CBA$ is parallel to the line $M_bP_b^1$;
(5) nine-point circle and incircle touch internally.

Primitive constructions:

(1) Given a point X and a line p one can construct a foot of the perpendicular from the point X to the line p;
(2) Given a point X and a circle k_1 one can construct a circle k_2 centered at point X which internally touches the circle k_1; NDG conditions are that the point X is inside the circle k_1 and that the point X is not the center of the circle k_1;
(3) Given a point A, a circle C, and a rational number r one can construct an image of the circle C in homothety with a center at the point A with coefficient r.