Automating Coherent Logic
an overview

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Status

- ACL = NFR 177562/V30, expiring 31.12.2012
  - Graduated: PhD Andrew Polonsky
  - PostDoc: Dag Hovland
- Collaboration (past, present, future)
  - Thierry Coquand (Chalmers)
  - Martin Giese, Bjarne Holen (UiO)
  - John Fisher (CalPolyTech, Pomona)
  - Hans de Nivelle (U Wroclaw)
  - Stefan Berghofer (TUM, Isabelle)
  - Hopefully you (...)
- Necessary: new impulses, initiatives

What has been achieved so far?

- Several CL-provers
- Some provers provide proof objects
- One prover (Geo) competitive in CASC 06-07 (FOF,FNT)
- A family of translations from FOL to CL
- Proof objects for the translation(s)
- Some integration in Isabelle (tactic coherent)
- About a dozen publications in various venues
- Ongoing: clp (RETE, Hovland), coColog (Fisher)

Challenges

- More efficient proof search in CL
- Evaluation of the translations FOL $\rightarrow$ CL
- Extensions with native equality
- Extensions with function symbols
- Tighter integration in proof assistants

Translation form FOL to CL

- Example: Peirce's Law $((p \to q) \to p) \to p$
- Variables $T, F : Prop \to Prop$ 'freezing the arguments', later expressing true and false
- Coherent theory:
  1. $F(((p \to q) \to p) \to p)$
  2. $F(((p \to q) \to p) \to p) \to T((p \to q) \to p) \wedge F(p)$
  3. $T((p \to q) \to p) \to F(p \to q) \vee T(p)$
  4. $F(p \to q) \to T(p) \wedge F(q)$
  5. $T(p) \wedge F(p) \to \bot$
- Refute in CL, substitute $T = \lambda p.\, p$ and $F = \lambda p.\, \neg p$ (on blackboard)

Example, continued

- $T((p \to q) \to p) \to F(p \to q) \vee T(p)$ was a *choice*
- Why not $T((p \to q) \to p) \wedge T(p \to q) \to T(p)$?
- Then you would need $F(p \to q) \vee T(p \to q)$ ...
- Sometimes, no disjunctions needed: $p \to q, p \vdash q$
- Polonsky's translation looks for the optimal polarities

Elimination of function symbols (de Nivelle)

- Unary predicates for constants: $C(x)$ for $c = x$, axiom $\exists x.\, C(x)$
- Binary predicates for unary functions: $F(x, y)$ for $f(x) = y$, axiom $\exists y.\, F(x, y)$
- And so on, unicity is not required!
- Example: for constants, $a = b$ becomes $A(x) \leftrightarrow B(x)$
- Equality almost vanishes: only needed between variables
- A combinatorial puzzle: each $n \in \mathbb{N}$ is either red or green but not both. For each $n \in \mathbb{N}$, if n is red then $n + 2$ is green else $n + 1$ is red. Is this possible?

Puzzle formalized in CL with functions

- $r(x) \to g(f(f(x)))$
- $g(x) \to r(f(x))$
- $r(x) \lor g(x)$
- $r(x) \land g(x) \to \bot$
- Domain non-empty!

Puzzle, functions eliminated

- $\exists y.\, F(x, y)$
- $r(x) \wedge F(x, y) \wedge F(y, z) \rightarrow g(z)$
- $g(x) \wedge F(x, y) \rightarrow r(y)$
- $r(x) \vee g(x)$
- $r(x) \wedge g(x) \rightarrow \bot$

Proof and recovery of proof object

- If necessary, translate FOL $\rightarrow$ CL
- Refute with prover
- Proofs are valid for all relations $F(x, y)$
- In particular for $F(x, y) \equiv (f(x) = y)$, substitute
- $\exists y. f(x) = y$ is a tautology
- $g(x) \wedge f(x) = y \rightarrow r(y)$ is equivalent to $g(x) \rightarrow r(f(x))$
- Similarly for all other axioms
- Proof of original formula is obtained

Queueing depth-first

- Observation: $\exists y.\, F(x, y)$ is harmful for depth-first search
- Recommended order for depth-first search:
  - Horn clauses clauses
  - Disjunctive clauses
  - Existential clauses
  - Disjunctive existential clauses
- Without functions, depth-first terminates for the first two
- Depth-first search not complete for one single existential clause, subtle: `p(a). p(b). q(b)->goal. p(X),p(Y)-> exists U: p(U),q(X),r(Y).`
- Queueing depth-first: the (disjunctive) existential clauses in a cyclic queue $+$ iterative deepening wrt constants. Complete.

Useful links

- General CL:
  `http://www.johnrfisher.net/index.html`
- RETE experiments:
  `http://code.google.com/p/clp/`
- Colog:
  `www.csupomona.edu/~jrfisher/colog2012/`