High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_1_01.) *Assuming that $A \notin p$ there exist point B, point C, such that $B \neq C$ and $B \in p$ and $C \in p$.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using ax_I3a).
2. The conclusion follows from the facts $B \neq C$ and $B \in p$ and $C \in p$.

QED

Theorem 2 (th_1_02.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ it holds that $\neg col(A, B, C)$.*

Proof:

1. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using ax_D1a).
2. From the fact $\neg col(B, C, A)$ it holds that $\neg col(B, A, C)$ and $\neg col(C, B, A)$ and $\neg col(C, A, B)$ and $\neg col(A, B, C)$ and $\neg col(A, C, B)$ (using ax_sym_ncol).
3. The conclusion follows from the fact $\neg col(A, B, C)$.

QED

Theorem 3 (th_1_03.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg col(A, B, C)$ there exist plane α, such that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane α, where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using ax_I4a).
2. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.

QED

Theorem 4 (th_1_04.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg col(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ and $A \in \alpha$.*

Proof:

1. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using ax_I6).
2. The conclusion follows from the facts $p \in \alpha$ and $A \in \alpha$.

QED
